VSM COLLEGE OF ENGINEERING

EEE DEPARTMENT

LECTURE NOTES

ON

FUNDAMENTALS OF ELECTRICAL MACHINES(R19 reg)

VSM COLLEGE OF ENGINEERING

RAMACHANDRAPURAM

III Year-II Semester **FUNDAMENTALS OF ELECTRICAL MACHINES**

UNIT -I:

Introduction

Active and passive elements- Ohm's Law – Kirchhoff's Laws –Electromagnetic Induction– Faraday's Laws - Series – Parallel circuits- Self and Mutual Inductance-Numerical problems. Purpose of Earthing – Methods of Earthing – Merits of Earthing. Different types of Electrical Machines.

UNIT -II:

DC Machines

Principle of operation of DC generator - Types of DC machines – EMF equation –- Open Circuit Characteristics- Principle of operation of DC Motor- Torque Equation- speed control methods of DC motor – Losses in DC machines - Swinburne's Test-Brake test on DC shunt motor – Performance Characteristics - Numerical problems.

UNIT -III:

Transformers

Principle of operation and construction Details – Classification of Transformers - EMF equation – Losses in a Transformer – Open Circuit & Short Circuit Test – Calculation of efficiency and regulation -Numerical Problems.

UNIT -IV:

Induction Motors

Principle of operation- Constructional Details - Classification – Revolving Magnetic Fields– Starting Methods – Numerical Problems. Principle of operation of Single Phase Induction Motor - Starting Methods- Applications.

UNIT -V:

Synchronous Machines

Principle of operation and construction of alternators –EMF Equation - Regulation of alternator by Synchronous Impedance Method – Numerical Problems.

Principle of operation of synchronous motor - Synchronous Condenser – Applications.

Text Books:

1. Principles of Electrical Machines by V.K. Mehta & Rohit Mehta, S.Chand publications
2. Theory & performance of Electrical Machines by J.B.Guptha, S.K.Kataria & Sons
3. Circuit Theory (Analysis and Synthesis) by A. Chakrabarti, Dhanpat Rai & Co.

Reference Books:

1. Basic Electrical Engineering by M.S.Naidu and S.Kamakshiah,TMH Publications
2. Fundamentals of Electrical Engineering by Rajendra Prasad, PHI Publications,2nd edition
3. Basic Electrical Engineering by Nagsarkar,Sukhija, Oxford Publications,2nd edition

Unit-I

Electrical Circuits

Introduction

1. Electric charge(Q)

In all atoms there exists number of electrons which are very loosely bounded to its nucleus. Such electrons are free to wander when specific forces are applied. If any of these electrons is removed, the atom becomes positively charged. And if excess electrons are added to the atom it becomes positively charged.

The total deficiency or addition of electrons in an atom is called its charge. A charged atom is called **Ion**. An element containing a number of ionized atoms is said to be charged. And accordingly the element consisting of that atom is said to be positively or negatively charged.

Particle	Electric charge possessed by particle of one number (C)	Atomic charge
Protons	$+1.6022X\ 10^{-19}$	+1
Neutrons	0	0
Electrons	$1.6022X\ 10^{-19}$	-1

The unit of measurement of charge is **Coulomb** (C). It can be defined as the charge possessed by number of electrons.

Hence if an element has a positive charge of one coulomb then that element has a deficiency of number of electrons.

Conductors:

The atoms of different materials differ in the number of electrons, protons and neutrons, which they contain. They also differ in how tightly the electrons in the outer orbit are bound to the nucleus. The electrons, which are loosely bound to their nuclei, are called **free electrons.** These free electrons may be dislodged from an atom by giving them additional energy. Thus they may be transferred from one atom to another. The electrical properties of materials largely depend upon the number of free electrons available.

A **conductor** is a material in which large number of free electrons is available. Thus current can flow easily through a conductor. All materials with resistivity less than 10^{-3} Ωm come under the category of conductors. Almost all metals are conductors.

Silver, copper Aluminum, carbon is some examples of conductors. Copper and Aluminum conductors are widely used in practice.

Insulators

Insulators are materials in which the outer electrons are tightly bound to the nucleus. It is very difficult to take out the electrons from their orbits. Consequently, current cannot flow through them. All materials with resistivity above 10^5 Ωm fall in the category of insulators.

Examples of insulators are mica, paper, glass, porcelain, rubber, oil and plastics.

Semiconductors

In some materials, the electrons in the outer orbits are normally held by the nucleus, but can be taken out by some means. These materials are called **semiconductors.**

Materials such as germanium and silicon are examples of semiconductors. The addition of slight traces of impurity to silicon or germanium can free the electrons. The semiconductors have resistivity between 10^{-3} and 10^5 Ωm.

Current(I)

An electric current is the movement of electric charges along a definite path. In case of a conductor the moving charges are electrons.

The unit of current in the International system of Units is the ***ampere (A)***.

The ampere is defined as that current which when flowing in two infinitely long parallel conductors of negligible cross-section, situated 1 meter apart in vacuum, produces between the conductors a force of 2×10^{-7} Newton per meter length.

Voltage(V)

Energy is required for the movement of charge from one point to another. Let W joules of energy be required to move positive charge of Q coulombs from *a* point a to *b* in a circuit. We say that a voltage exists between the two points.

The voltage across two terminals is a measure of the work required to move charge through the element. The unit of voltage is the volt, and 1 volt is the same as 1 J/C. Voltage is represented by V or *v*.

The voltage V between two points may be defined in terms of energy that would be required if a charge were transferred from one point to the other. A voltage can exist between the two electrical terminals whether a current is flowing or not.

Voltage between *a* and *b* is given by

$$V = \frac{W}{Q} \ (J / C)$$

Electromotive Force (EMF)

The emf represents the driving influence that causes a current to flow, and may be interpreted to represent the energy that is used during passing of a unit charge through the source. The term emf is always associated with energy conversion. The emf is usually represented by the symbol E and has the unit **VOLT**

If W = energy imparted by the voltage source in joules (J)
Q = charge transferred through the source in coulombs ©
E = e.m.f. of the source

Then

$$E = \frac{W}{Q} (J / C)$$

Potential Difference:

The potential difference (p.d.) between two points is the energy required to move one coulomb of charge from one to the other.

If : W = energy required to transfer the charge
Q = charge transferred between the points
V = potential difference

Then

$$V = \frac{W}{Q}(J / C)$$

Electric Power

Power is the rate at which work is done. Work is done whenever a force causes motion. If a mechanical force is applied to lift or move a weight, work is done. We know that voltage is an electric force and it forces current to flow in a circuit. When voltage causes current flow (electrons to move), work is done.

The rate at which is work is done is called electric power and is measured in **watts**.

$$P = \frac{W}{t}(J / s)$$

Basic Circuit Components

Resistor, inductor, and capacitor are the three basic components of a network. A resistor is an element that dissipates energy as heat when current passes through it. An inductor stores energy by virtue of a current through it. A capacitor stores energy by virtue of a voltage existing across it.

Resistance

The opposition offered by a substance to the flow of electric current is called **resistance**. Since current is the flow of free electrons, resistance is the opposition offered by the substance to flow of free electrons. This opposition occurs because atoms and molecules of the substance obstruct the flow of these electrons. Certain substances (e.g., metals such as silver, copper, aluminum etc) offer very little opposition to the flow of electric current and are called conductors. On the other hand, those substances which offer high opposition to the flow of electric current (i.e., flow of free electrons) are called insulators e.g., glass, rubber, mica, dry wood etc.

It may be noted here that resistance is the electric friction offered by the substance and causes production of heat with the flow of electric current. The moving electrons collide with atoms or molecules of the substance ; each collision resulting in the liberation of minute quantity of heat.

- Denoted by **R**
- Unit is **Ohms(Ω)**
- Symbol :

Unit of resistance: The practical unit of resistance is ohm and is represented by the symbol Ω. It is defined as under:
A wire is said to have a resistance of **1 ohm** *if a p.d. of 1 volt across its ends causes 1 ampere of current to flow through it.*

Factors upon which Resistance Depends:

The resistance R of a conductor

(i) is directly proportional to its length (*l*)
(ii) is inversely proportional to its area of cross-section (a)
(iii) Depends upon the nature of material.
(iv) Changes with temperature.

From the first three points (leaving temperature for the time being), we have,

$$R \propto \frac{l}{a} \quad \text{or} \quad R = \rho \frac{l}{a}$$

Where ρ (Greek letter 'Rho') is a constant and is known as resistivity or specific resistance of the material. Its value depends upon the nature of material.

Material	Resistivity ρ (ohm m)
Copper	1.68×10^{-8}
Aluminum	2.65×10^{-8}
Tungsten	5.60×10^{-8}

Conductance:

Conductance (G) is the reciprocal of resistance.

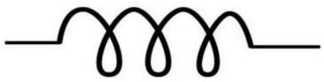
We know $$G = \frac{1}{R} = \frac{1}{\rho}\frac{A}{l} \quad \textbf{or} \quad G = \sigma \frac{A}{l}$$

Where σ (Greek letter 'sigma') is called the conductivity or specific conductance of the material. The unit of conductance is mho.

<u>Inductors</u>

The electrical element that stores energy in association with a flow of current is called *inductor* . The idealized circuit model for the inductor is called an *inductance.* Practical inductors are made of many turns of thin wire wound on a magnetic core or an air core. A unique feature of the inductance is that its presence in a circuit is felt only when there is a changing current. Fig below shows a schematic representation of an inductor.

- Denoted as L.
- Units: Henry (H).
- Symbol



Schematic representation of an inductor

For the ideal circuit model of an inductor, the voltage across it is proportional to the rate of change of current in it. Thus if the rate of change of current is di/dt and *v* is the induced voltage, then

$$v \;\alpha\; \frac{di}{dt} \quad \text{or} \quad v = L\frac{di}{dt} \text{ volt} \qquad \text{(Eq – 1)}$$

In the above equation proportionality constant L is called inductance. The unit of inductance is Henry, named after the American physicist Joseph Henry. Equation may be rewritten as

$$L = \frac{v}{\frac{di}{dt}} = \frac{volt - \sec ond}{ampere} \quad \text{or Henrys (H)} \qquad \text{(Eq – 2)}$$

If an inductor induces a voltage of 1 V when the current is uniformly varying at the rate of 1 A/sec, it is said to have an inductance of 1 H. Integrating Eq – 1 with respect to time t,

$$i = \frac{1}{L}\int_0^t vdt + i(0) \qquad \text{(Eq –3)}$$

Where $i(0)$ is the current at t = 0. From Eq – 3 it may be inferred that the current in an inductor cannot change suddenly in zero time.

Instantaneous power *p* entering the inductor at any instant is given by

$$p = vi = Li\frac{di}{dt} \qquad \text{(Eq – 4)}$$

When the current is constant, the derivative is zero and no additional energy is stored in the inductor. When the current increases, the derivative is positive and hence the power is positive; and, in turn, an additional energy is stored in the inductor. The energy stored in the inductor, W_L, is given by

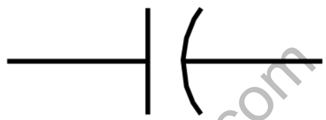
$$W_L = \int_0^t vidt = \int_0^t Li\frac{di}{dt}dt = L\int_0^t idi = \frac{1}{2}Li^2 \text{ Joule} \qquad \text{(Eq – 5)}$$

Eq – 5 assumes that the inductor has no previous history, that is, at $t = 0,\ i = 0.$ the energy is stored in the inductor in a magnetic field. When the current increases, the stored energy is the magnetic field also increases. When the current reduces to zero, the energy stored in the inductor is returned to the source from which it receives the energy.

<u>Capacitors:</u>

A *capacitor* is a device that can store energy in the form of a charge separation when it is suitably polarized by an electric field by applying a voltage across it. In the simplest form, a capacitor consists of two parallel conducting plates separated by air or any insulating material, such as mica. It has the characteristic of storing electric energy (charge), which can be fully retrieved, in an electric field. A significant feature of the capacitor is that its presence is felt in an electric circuit when a changing potential difference exists across the capacitor. The presence of an insulating material between the conducting plates does not allow the flow of dc current; thus a capacitor acts as an open circuit in the presence of dc current.

- Denoted as C
- Unit is Farad (F).
- Symbol



(Schematic representation of a capacitor.)

The ability of the capacitor is store charge is measured in terms of capacitance C. *Capacitance* of a capacitor is defined as charge stored per volt applied and its unit is farad (F). However, for practical purposes the unit of farad is too large. Hence, microfarad (μ F) is used to specify the capacitance of the components and circuits.

If the charge on the capacitor at any time t after the switch S is closed in q coulombs and the voltage across it is v volts. Then by definition

$$\boxed{C = \frac{q}{v}} \text{ coulomb} \qquad \text{(Eq – 6)}$$

Current i flowing through the capacitor can be obtained as

$$\boxed{i = \frac{dq}{dt} = C\frac{dv}{dt}} \text{ Ampere} \qquad \text{(Eq – 7)}$$

Eq – 7 is integrated with respect to time to get the voltage across the capacitor as

$$\boxed{v = \frac{1}{C}\int_0^t idt + v(0)} \qquad \text{(Eq – 8)}$$

Where $v(0)$ denotes the initial voltage across the capacitor at t = 0.

Power p in the capacitor is given as

$$p = vi = Cv\frac{dv}{dt} \text{ Watts} \qquad \text{(Eq – 9)}$$

Energy stored in capacitor, W_C, is given by

$$W_C = \int p\,dt = C\int v\,dv = \frac{1}{2}Cv^2 \text{ joule} \qquad \text{(Eq – 10)}$$

Capacitance of a capacitor depends on its dimensions :

A capacitor consists of two electrodes (plates) separated by a insulating material (dielectric). If the area of the plate in A m^2 and the distance between them is d m, it is observed that

$C \propto A$ and $C \propto \frac{1}{d}$

$$\therefore \quad C = \frac{\varepsilon A}{d} \qquad \text{(Eq – 11)}$$

Where ε is the absolute permittivity constant. The absolute permittivity constant depends on the type of dielectric employed in the capacitor. The ratio of the absolute permittivity constant of the dielectric ε to the permittivity constant of vacuum ε_0 is called relative permittivity ε_r, that is,

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

Hence,

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

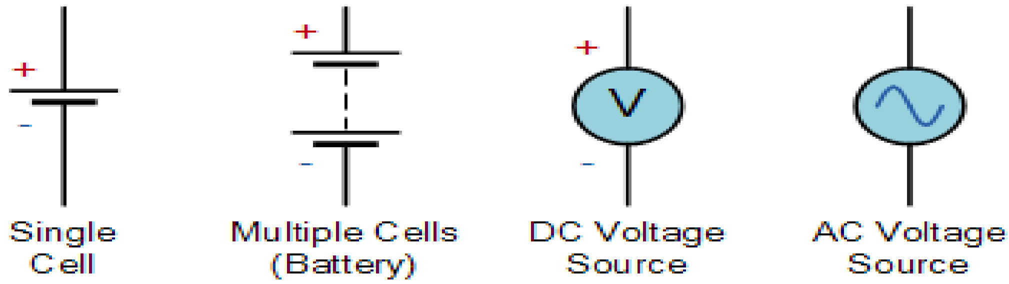
The units for absolute permittivity ε can be established from Eq.11 as under:

$$\varepsilon = \frac{C(\,farads) \times d\,(meters)}{A(meters)^2} = \frac{C \times d}{A} \quad (\,farads\ /\ metre(F\ /\ m)\ )$$

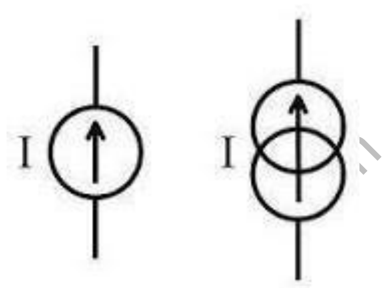
Based on experimental results, the value of the permittivity constant of vacuum has been found to be equal to 8.85×10^{-12} F/m. Therefore, the value of ε_r for vacuum is 1.0 and for air is 1.0006. For practical purposes, the value of ε_r for air is also taken as 1.

VOLTAGE SOURCE

It is a two terminal device which can maintain a fixed voltage. An ideal voltage source can maintain the fixed voltage independent of the load resistance or the output current. However, a real-world voltage source cannot supply unlimited current. A voltage source is the dual of a current source. Real-world sources of electrical energy, such as batteries, generators, and power systems, can be modeled for analysis purposes as a combination of an ideal voltage source.



CURRENT SOURCE -is an electronic circuit that delivers or absorbs an electric current which is independent of the voltage across it. A current source is the dual of a voltage source.The term constant-current 'sink' is sometimes used for sources fed from a negative voltage supply. Figure 1 shows the schematic symbol for an ideal current source, driving a resistor load There are two types - an independent current source (or sink) delivers a constant current. A dependent current source delivers a current which is proportional to some other voltage or current in the circuit.



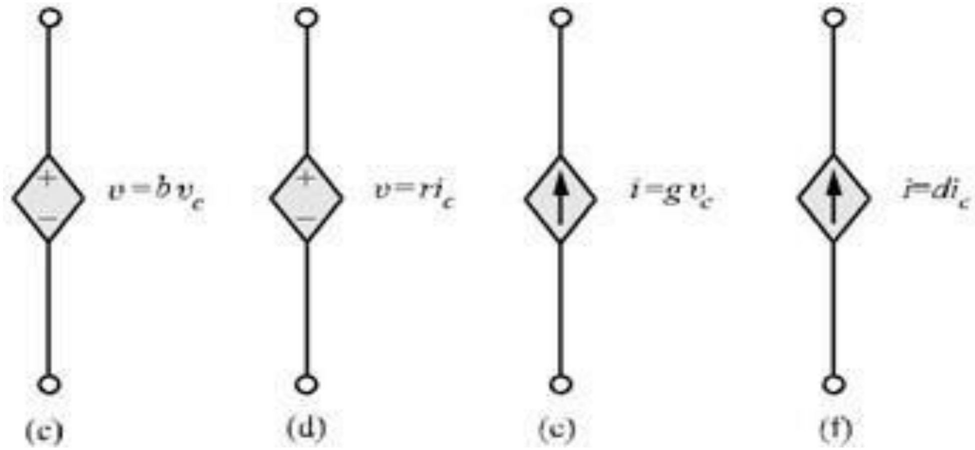
Independent Sources -The ideal voltage source and ideal current source discussed here come under the category of *independent sources*. The independent source is one which does not depend on any other quantity in the circuit. It has a constant value i.e., the strength of voltage or current is not changed by any variation in the connected circuit. Thus, the voltage or current is fixed and is not adjustable.

Dependent Sources: The source whose output voltage or current is not fixed but depends on the voltage or current in another part of the circuit is called as dependent or controlled source.

The dependent source is basically a three terminal device. The three terminals are paired with one common terminal. One pair is referred as input while the other pair as output. For example, in a transistor, the output voltage depends upon the input voltage. The dependent sources are represented by diamond shaped box as shown in Fig.

The dependent sources can be categorized as:

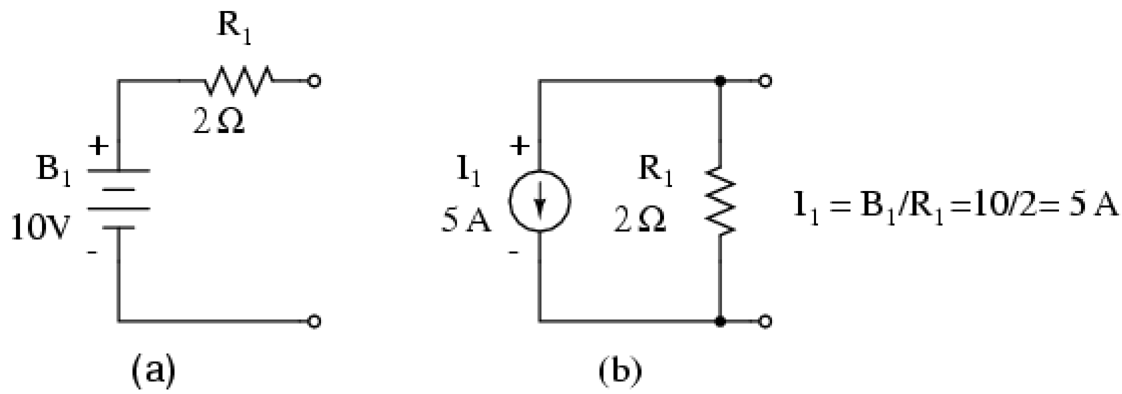
c. Voltage dependent voltage source
d. Current dependent voltage source
e. Voltage dependent current source
f. Current dependent current source



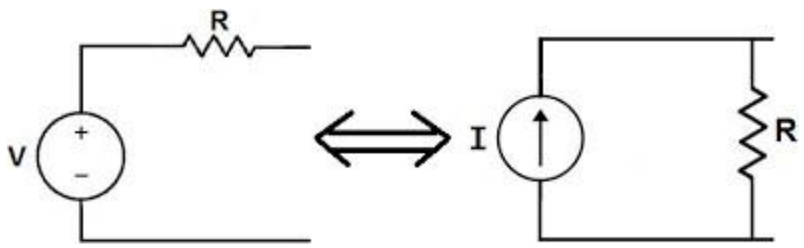
SOURCE CONVERSION

It is most important part of the circuit analysis. To simplify the circuit, certain rules have been framed which are given below:

(a) A voltage source having some resistance can be replaced by the current source in parallel with the resistance as shown in Fig.



(b) A current source in parallel with some resistance can be replaced by a voltage source in series with the same resistance as shown in fig.



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Ohms Law

The relationship between Voltage, Current and Resistance in any DC electrical circuit was firstly discovered by the German physicist **Georg Ohm**.**Ohm** found that, at a constant temperature, the electrical current flowing through a fixed linear resistance is directly proportional to the voltage applied across it, and also inversely proportional to the resistance.

Ohms Law Relationship

$$\text{Current, } (I) = \frac{\text{Voltage, } (V)}{\text{Resistance, } (R)} \text{ in Amperes, } (A)$$

By knowing any two values of the Voltage, Current or Resistance quantities we can use **Ohms Law** to find the third missing value. **Ohms Law** is used extensively in electronics formulas and calculations so it is "very important to understand and accurately remember these formulas"

To find the Voltage, (V)

[V = I x R](volts) = I (amps) x R (Ω)

To find the Current, (I)

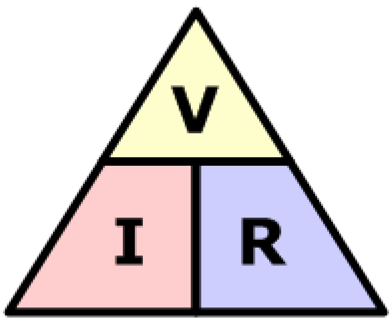
[$I = \frac{V}{R}$] I (amps) = V (volts) ÷ R (Ω)

To find the Resistance, (R)

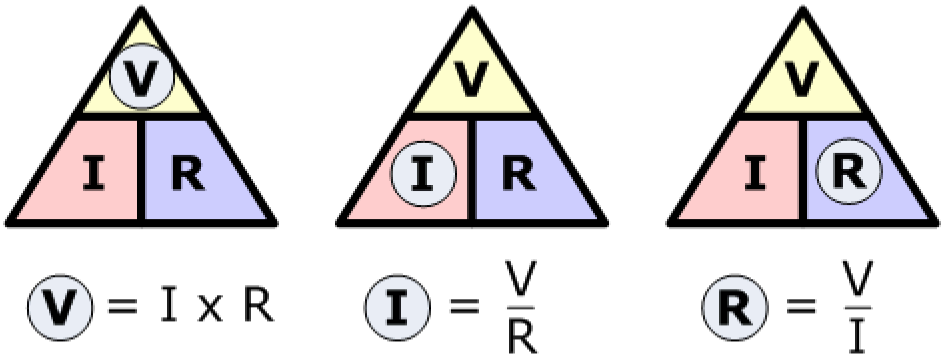
[$R = \frac{V}{I}$] R (Ω) = V (volts) ÷ I (amps)

It is sometimes easier to remember this Ohms law relationship by using pictures. Here the three quantities of V, I and R have been superimposed into a triangle (affectionately called the **Ohms Law Triangle**) giving voltage at the top with current and resistance below. This arrangement represents the actual position of each quantity within the Ohms law formulas.

Ohms Law Triangle

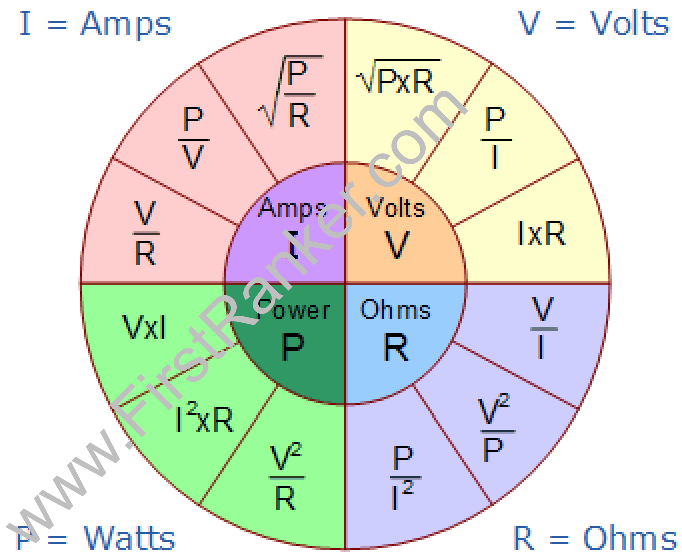


Transposing the standard Ohms Law equation above will give us the following combinations of the same equation:



Then by using Ohms Law we can see that a voltage of 1V applied to a resistor of 1Ω will cause a current of 1A to flow and the greater the resistance value, the less current that will flow for a given applied voltage. Any Electrical device or component that obeys "Ohms Law" that is, the current flowing through it is proportional to the voltage across it (I α V), such as resistors or cables, are said to be **"Ohmic"** in nature, and devices that do not, such as transistors or diodes, are said to be **"Non-ohmic"** devices.

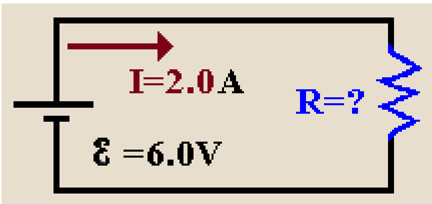
Ohms Law Pie Chart



Ohms Law Matrix Table

Ohms Law Formulas				
Known Values	Resistance (R)	Current (I)	Voltage (V)	Power (P)
Current & Resistance	---	---	V = IxR	$P = I^2xR$
Voltage & Current	$R = \frac{V}{I}$	---	---	P = VxI
Power & Current	$R = \frac{P}{I^2}$	---	$V = \frac{P}{I}$	---
Voltage & Resistance	---	$I = \frac{V}{R}$	---	$P = \frac{V^2}{R}$
Power & Resistance	---	$I = \sqrt{\frac{P}{R}}$	$V = \sqrt{PxR}$	---
Voltage & Power	$R = \frac{V^2}{P}$	$I = \frac{P}{V}$	---	---

Question 1:
An emf source of 6.0V is connected to a purely resistive lamp and a current of 2.0 amperes flows. All the wires are resistance-free. What is the resistance of the lamp?



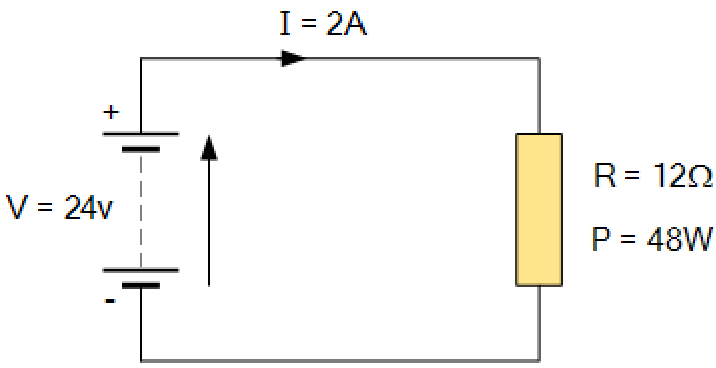
Solution: The gain of potential energy occurs as a charge passes through the battery, that is, it gains a potential of E =6.0V. No energy is lost to the wires, since they are assumed to be resistance-free. By conservation of energy, the potential that was gained (i.e. =V=6.0V) must be lost in the resistor. So, by Ohm's Law:

$$V = I R$$

$$R = \frac{V}{I}$$

$$R = 3.0\ \Omega$$

Problem 2: For the circuit shown below find the Voltage (V), the Current (I), the Resistance (R) and the Power (P).



Voltage [V = I x R] = 2 x 12Ω = 24V

Current [I = V ÷ R] = 24 ÷ 12Ω = 2A

Resistance [R = V ÷ I] = 24 ÷ 2 = 12 Ω

Power [P = V x I] = 24 x 2 = 48W

Power within an electrical circuit is only present when **BOTH** voltage and current are present. For example, in an open-circuit condition, voltage is present but there is no current flow I = 0 (zero), therefore V x 0 is 0 so the power dissipated within the circuit must also be 0. Likewise, if we have a short-circuit condition, current flow is present but there is no voltage V = 0, therefore 0 x I = 0 so again the power dissipated within the circuit is 0.

As electrical power is the product of V x I, the power dissipated in a circuit is the same whether the circuit contains high voltage and low current or low voltage and high current flow. Generally, electrical power is dissipated in the form of **Heat** (heaters), **Mechanical Work** such as motors, **Energy**in the form of radiated (Lamps) or as stored energy (Batteries).

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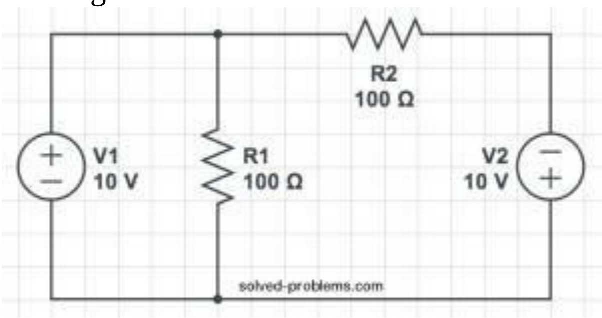
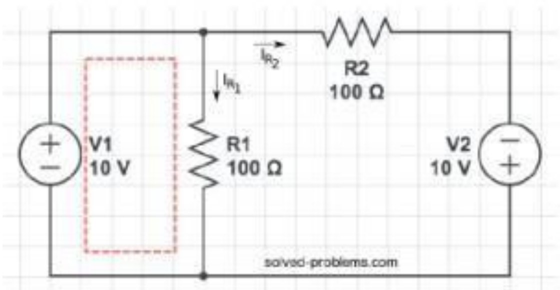
Problem:

Find resistor currents using KVL.



Solution:

R_1 and V_1 are parallel. So the voltage across R_1 is equal to V_1 . This can be also calculated using KVL in the left hand side loop:

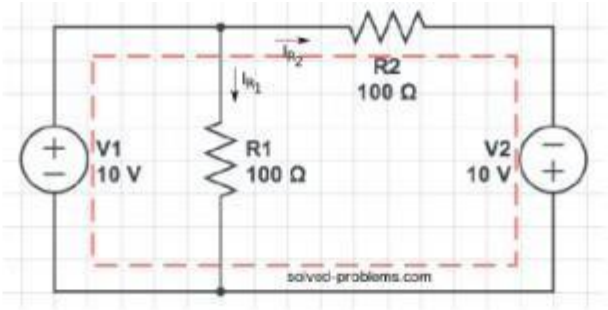


$-V_1 + V_{R_1} = 0 \rightarrow V_{R_1} = V_1 = 10V$.

Now, use Ohm's law to find I_{R_1} :

$V_{R_1} = R_1 \times I_{R_1} \rightarrow I_{R_1} = \frac{V_{R_1}}{R_1} = 0.1A$.

To find I_{R_2} , write KVL around the outer loop:



$$-V_1 + V_{R_2} - V_2 = 0 \rightarrow V_{R_2} = V_1 + V_2 = 20V .$$

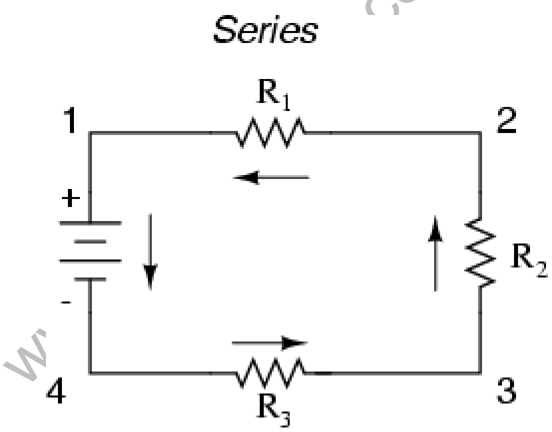
Again, use Ohm's law to determine I_{R_2} :

$$V_{R_2} = R_2 \times I_{R_2} \rightarrow I_{R_2} = \frac{V_{R_2}}{R_2} = 0.2A .$$

Series and Parallel Circuits

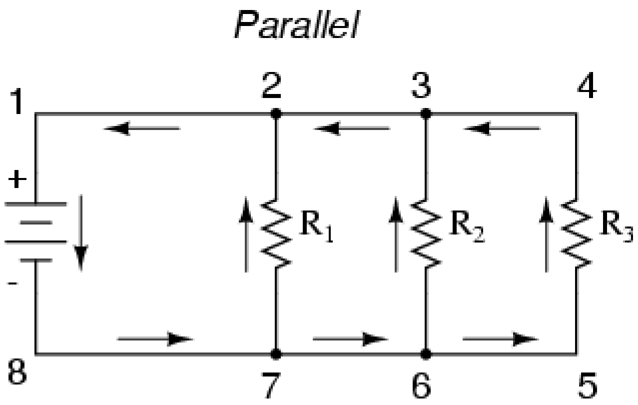
Circuits consisting of just one battery and one load resistance are very simple to analyze, but they are not often found in practical applications. Usually, we find circuits where more than two components are connected together.

There are two basic ways in which to connect more than two circuit components: *series* and *parallel*. First, an example of a series circuit:



Here, we have three resistors (labeled R_1, R_2, and R_3), connected in a long chain from one terminal of the battery to the other. (It should be noted that the subscript labeling—those little numbers to the lower-right of the letter "R"—are unrelated to the resistor values in ohms. They serve only to identify one resistor from another.) The defining characteristic of a series circuit is that there is only one path for electrons to flow. In this circuit the electrons flow in a counter-clockwise direction, from point 4 to point 3 to point 2 to point 1 and back around to 4.

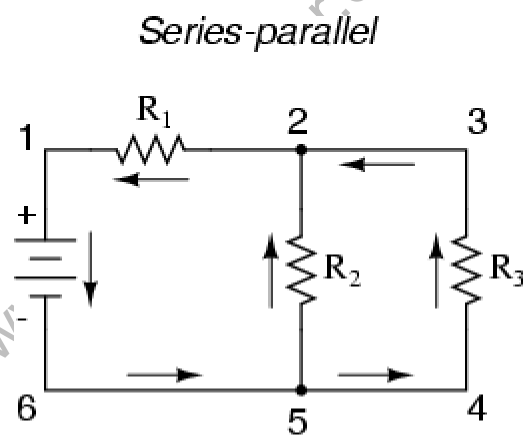
Now, let's look at the other type of circuit, a parallel configuration:



Again, we have three resistors, but this time they form more than one continuous path for electrons to flow. There's one path from 8 to 7 to 2 to 1 and back to 8 again. There's another from 8 to 7 to 6 to 3 to 2 to 1 and back to 8 again. And then there's a third path from 8 to 7 to 6 to 5 to 4 to 3 to 2 to 1 and back to 8 again. Each individual path (through R_1, R_2, and R_3) is called a *branch*.

The defining characteristic of a parallel circuit is that all components are connected between the same set of electrically common points. Looking at the schematic diagram, we see that points 1, 2, 3, and 4 are all electrically common. So are points 8, 7, 6, and 5. Note that all resistors as well as the battery are connected between these two sets of points.

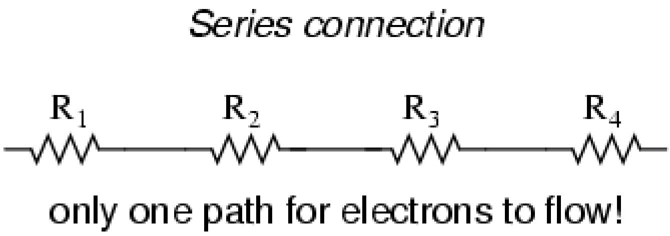
And, of course, the complexity doesn't stop at simple series and parallel either! We can have circuits that are a combination of series and parallel, too:



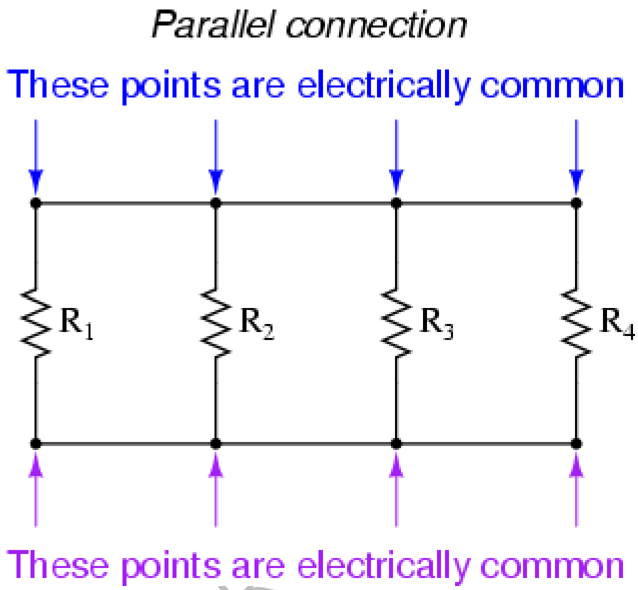
In this circuit, we have two loops for electrons to flow through: one from 6 to 5 to 2 to 1 and back to 6 again, and another from 6 to 5 to 4 to 3 to 2 to 1 and back to 6 again. Notice how both current paths go through R_1 (from point 2 to point 1). In this configuration, we'd say that R_2 and R_3 are in parallel with each other, while R_1 is in series with the parallel combination of R_2 and R_3.

This is just a preview of things to come. Don't worry! We'll explore all these circuit configurations in detail, one at a time!

The basic idea of a "series" connection is that components are connected end-to-end in a line to form a single path for electrons to flow:



The basic idea of a "parallel" connection, on the other hand, is that all components are connected across each other's leads. In a purely parallel circuit, there are never more than two sets of electrically common points, no matter how many components are connected. There are many paths for electrons to flow, but only one voltage across all components:

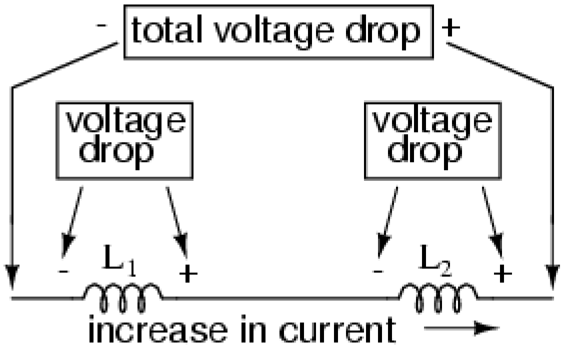


Series and parallel resistor configurations have very different electrical properties. We'll explore the properties of each configuration in the sections to come.

- **REVIEW:**
- In a series circuit, all components are connected end-to-end, forming a single path for electrons to flow.
- In a parallel circuit, all components are connected across each other, forming exactly two sets of electrically common points.
- A "branch" in a parallel circuit is a path for electric current formed by one of the load components (such as a resistor).

Inductors in Series and Parallel:

When inductors are connected in series, the total inductance is the sum of the individual inductors' inductances. To understand why this is so, consider the following: the definitive measure of inductance is the amount of voltage dropped across an inductor for a given rate of current change through it. If inductors are connected together in series (thus sharing the same current, and seeing the same rate of change in current), then the total voltage dropped as the result of a change in current will be additive with each inductor, creating a greater total voltage than either of the individual inductors alone. Greater voltage for the same rate of change in current means greater inductance.

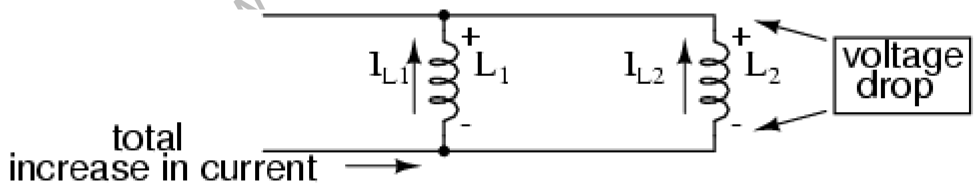


Thus, the total inductance for series inductors is more than any one of the individual inductors' inductances. The formula for calculating the series total inductance is the same form as for calculating series resistances:

Series Inductances

$$L_{total} = L_1 + L_2 + \ldots L_n$$

When inductors are connected in parallel, the total inductance is less than any one of the parallel inductors' inductances. Again, remember that the definitive measure of inductance is the amount of voltage dropped across an inductor for a given rate of current change through it. Since the current through each parallel inductor will be a fraction of the total current, and the voltage across each parallel inductor will be equal, a change in total current will result in less voltage dropped across the parallel array than for any one of the inductors considered separately. In other words, there will be less voltage dropped across parallel inductors for a given rate of change in current than for any of those inductors considered separately, because total current divides among parallel branches. Less voltage for the same rate of change in current means less inductance.



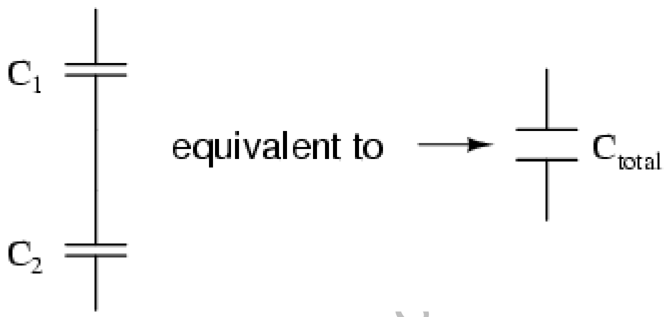
Thus, the total inductance is less than any one of the individual inductors' inductances. The formula for calculating the parallel total inductance is the same form as for calculating parallel resistances:

Parallel Inductances

$$L_{total} = \cfrac{1}{\cfrac{1}{L_1} + \cfrac{1}{L_2} + \ldots \cfrac{1}{L_n}}$$

Capacitors in series and parallel:

When capacitors are connected in series, the total capacitance is less than any one of the series capacitors' individual capacitances. If two or more capacitors are connected in series, the overall effect is that of a single (equivalent) capacitor having the sum total of the plate spacings of the individual capacitors. As we've just seen, an increase in plate spacing, with all other factors unchanged, results in decreased capacitance.

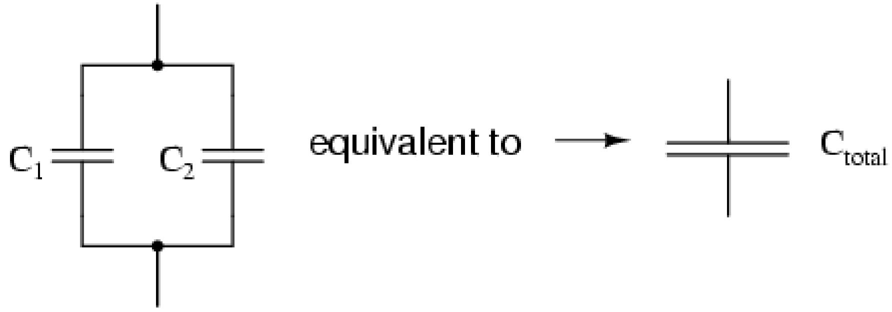


Thus, the total capacitance is less than any one of the individual capacitors' capacitances. The formula for calculating the series total capacitance is the same form as for calculating parallel resistances:

Series Capacitances

$$C_{total} = \cfrac{1}{\cfrac{1}{C_1} + \cfrac{1}{C_2} + \ldots \cfrac{1}{C_n}}$$

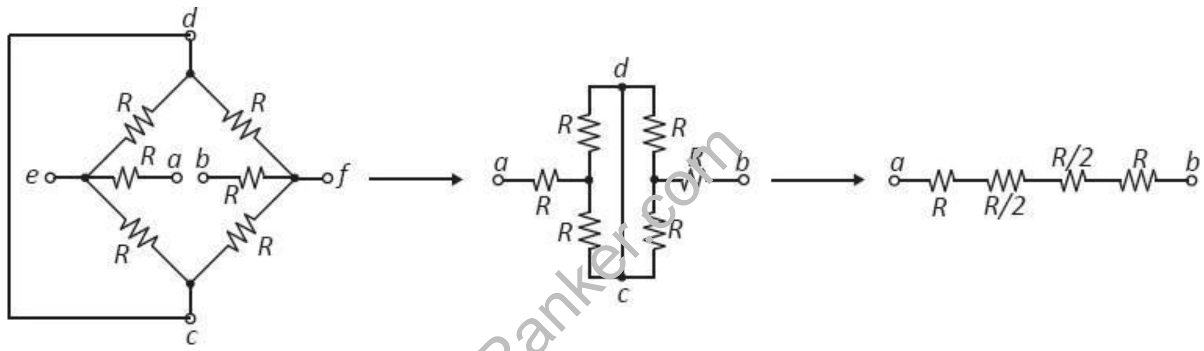
When capacitors are connected in parallel, the total capacitance is the sum of the individual capacitors' capacitances. If two or more capacitors are connected in parallel, the overall effect is that of a single equivalent capacitor having the sum total of the plate areas of the individual capacitors. As we've just seen, an increase in plate area, with all other factors unchanged, results in increased capacitance.

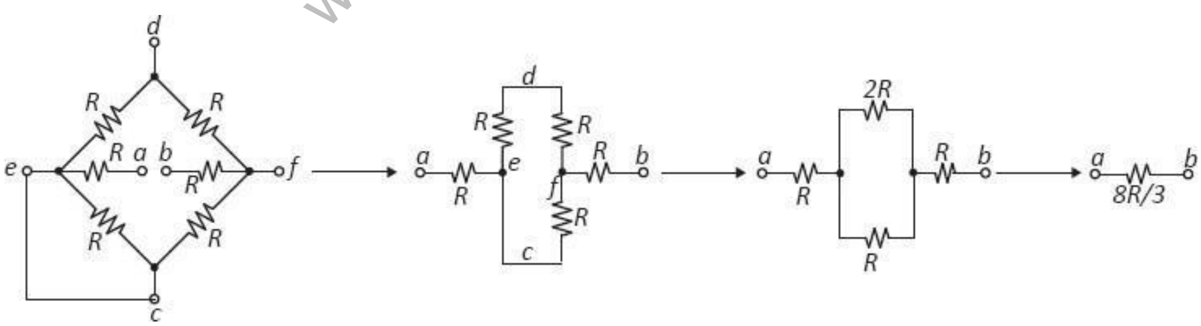


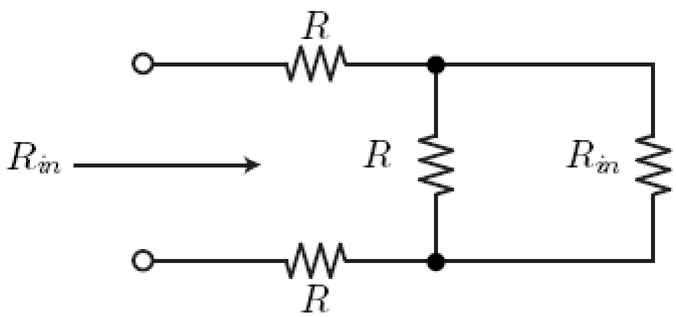
Thus, the total capacitance is more than any one of the individual capacitors' capacitances. The formula for calculating the parallel total capacitance is the same form as for calculating series resistances:

Parallel Capacitances

$$C_{total} = C_1 + C_2 + \ldots C_n$$







Introduction to PORT NETWORKS

There are certain circuit configurations that cannot be simplified by series-parallel combination alone. A simple transformation based on mathematical technique is readily simplifies the electrical circuit configuration. A circuit configuration shown below

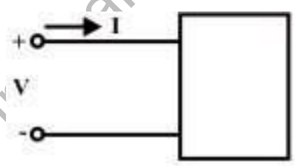


Fig. 6.1(a) One port network

is a general **one-port circuit.** When any voltage source is connected across the terminals, the current entering through any one of the two terminals, equals the current leaving the other terminal. For example, resistance, inductance and capacitance acts as a **one-port.** On the other hand, a **two-port** is a circuit having two pairs of terminals. Each pair behaves as a one-port; current entering in one terminal must be equal to the current living the other terminal.

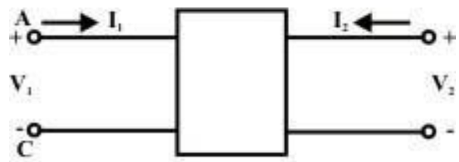


Fig. 6.1(b) Two port network

Fig.6.1.(b) can be described as a four terminal network, for convenience subscript 1 to refer to the variables at the input port (at the left) and the subscript 2 to refer to the variables at the output

port (at the right). The most important subclass of two-port networks is the one in which the minus reference terminals of the input and output ports are at the same. This circuit configuration is readily possible to consider the ‘π’ or ‘Δ’ – network also as a three-terminal network in fig.6.1(c). Another frequently encountered circuit configuration that shown in fig.6.1(d) is approximately referred to as a three-terminal *Y* connected circuit as well as two-port circuit.

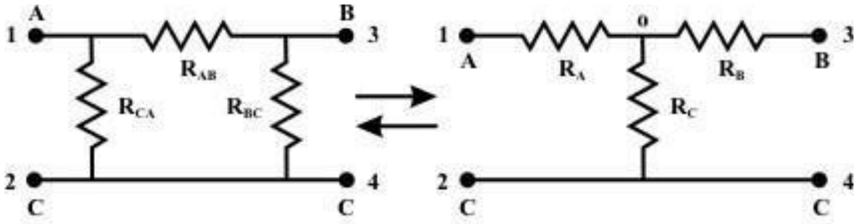


Fig. 6.1 (c) Fig. 6.1 (d)

The name derives from the shape or configuration of the circuit diagrams, which look respectively like the letter Y and the Greek capital letter Δ.

Delta – Wye conversion (Δ) (Y)

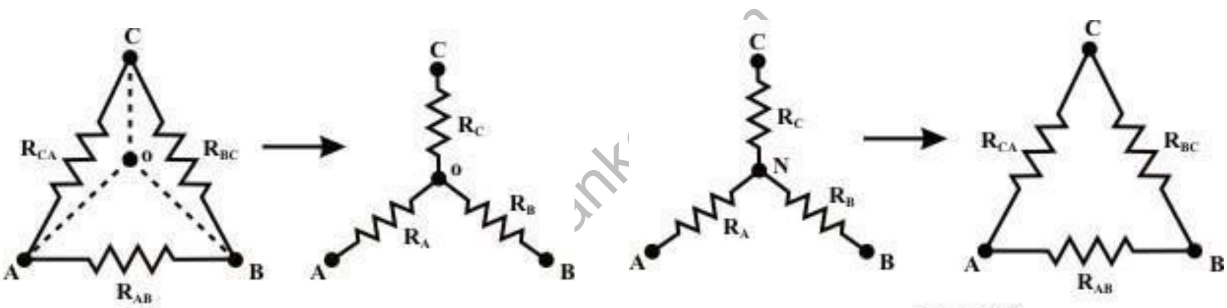


Fig. 6.1 (e) Fig. 6.1 (f)

These configurations may often be handled by the use of a Δ−*Y* or −Δ*Y* transformation. One of the most basic three-terminal network equivalent is that of three resistors connected in “Delta” Δ and in “Wye(Y)”. These two circuits identified in fig.L6.1 (e) and Fig.L.6.1 (f) are sometimes part of a larger circuit and obtained their names from their configurations. These three terminal networks can be redrawn as four-terminal networks as shown in fig.L.6.1(c) and fig.L.6.1 (d). We can obtain useful expression for direct transformation or conversion from Δ to *Y* or *Y* to Δ by considering that for equivalence the two networks have the same resistance when looked at the similar pairs of terminals.

Conversion from Delta (Δ) to Star or Wye (Y)

Let us consider the network shown in fig.6.1(e) and assumed the resistances (*RAB*,*RBC and RCA*) in Δ network are known. Our problem is to find the values of in Wye (*Y*) network (see fig.6.1(e)) that will produce the same resistance when measured between similar pairs of terminals. We can write the equivalence resistance between any two terminals in the following form.

Between A & C terminals:

$$R_A + R_C = \frac{R_{CA}\left(R_{AB} + R_{BC}\right)}{R_{AB} + R_{BC} + R_{CA}}$$

Between C & B terminals:

$$R_C + R_B = \frac{R_{BA}\left(R_{AB} + R_{CA}\right)}{R_{AB} + R_{BC} + R_{CA}}$$

Between B & A terminals:

$$R_B + R_A = \frac{R_{AB}\left(R_{CA} + R_{BC}\right)}{R_{AB} + R_{BC} + R_{CA}}$$

By combining above three equations, one can write an expression as given below.

$$R_A + R_B + R_C = \frac{R_{AB}R_{BC} + R_{BC}R_{CA} + R_{CA}R_{AB}}{R_{AB} + R_{BC} + R_{CA}}$$

we can write the express for unknown resistances of Wye (Y) network as

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_B = \frac{R_{AB}R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_C = \frac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

When we need to transform a Delta (Δ) network to an equivalent Wye (Y) network, the equations (6.5) to (6.7) are the useful expressions. On the other hand, the equations (6.12) – (6.14) are used for Wye (Y) to Delta (Δ) conversion.

<u>Conversion from Star(Y) or Wye to Delta (Δ)</u>

To convert a **Wye (**Y) to a **Delta (**Δ), the relationships R_{AB}, R_{BC} *and* R_{CA} must be obtained in terms of the **Wye** resistances R_A R_B *and* R_C (referring to fig.6.1 (f)). Considering the Y connected network, we can write the current expression through *RA* resistor as

$$I_A = \frac{\left(V_A - V_N\right)}{R_A} \quad \text{(for } Y \text{ network)}$$

Appling KCL at '' for Y connected network (assume N,,ABC terminals having higher potential than the terminal) we have,

$$\frac{(V_A - V_N)}{R_A} + \frac{(V_B - V_N)}{R_B} + \frac{(V_C - V_N)}{R_C} = 0 \quad \Rightarrow V_N\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right) = \left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)$$

or, $$\Rightarrow V_N = \frac{\left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)}$$

For Δ-network (see fig.6.1.(f)),

Current entering at terminal A = Current leaving the terminal 'A'

$$I_A = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \text{ (for } \Delta \text{ network)} \tag{6.10}$$

From equations (6.8) and (6.10),

$$\frac{(V_A - V_N)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

Using the V_N expression in the above equation, we get

$$\frac{\left(V_A - \frac{\left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)}\right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \quad \Rightarrow \quad \frac{\left(\frac{\left(\frac{V_A - V_B}{R_B} + \frac{V_A - V_C}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)}\right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

or $$\frac{\left(\frac{\left(\frac{V_{AB}}{R_B} + \frac{V_{AC}}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)}\right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \tag{6.11}$$

Equating the coefficients of V_{AB} and V_{AC} in both sides of eq.(6.11), we obtained the following relationship.

$$\frac{1}{R_{AB}} = \frac{1}{R_A R_B\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)} \quad \Rightarrow R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$\frac{1}{R_{AC}} = \frac{1}{R_A R_C\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)} \quad \Rightarrow R_{AC} = R_A + R_C + \frac{R_A R_C}{R_B}$$

Similarly, I_B for both the networks (see fig.61(f)) are given by

$$I_B = \frac{(V_B - V_N)}{R_B} \quad \text{(for } Y \text{ network)}$$

$$I_B = \frac{V_{BC}}{R_{BC}} + \frac{V_{BA}}{R_{BA}} \quad \text{(for } \Delta \text{ network)}$$

Equating the above two equations and using the value of V_N (see eq.(6.9), we get the final expression as

$$\frac{\left(\frac{\left(\frac{V_{BC}}{R_C}+\frac{V_{BA}}{R_A}\right)}{\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)}\right)}{R_B} = \frac{V_{BC}}{R_{BC}} + \frac{V_{BA}}{R_{BA}}$$

Equating the coefficient of V_{BC} in both sides of the above equations we obtain the following relation

$$\frac{1}{R_{BC}} = \frac{1}{R_B R_C\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)} \quad \Rightarrow R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A} \qquad (6.14)$$

When we need to transform a Delta (Δ) network to an equivalent Wye (*Y*) network, the equations (6.5) to (6.7) are the useful expressions. On the other hand, the equations (6.12) – (6.14) are used for Wye (*Y*) to Delta (Δ) conversion.

2. D.C. GENERATORS

Introduction

Although a far greater percentage of the electrical machines in service are a.c. machines, the d.c. machines are of considerable industrial importance. The principal advantage of the d.c. machine, particularly the d.c. motor, is that it provides a fine control of speed. Such an advantage is not claimed by any a.c. motor. However, d.c. generators are not as common as they used to be, because direct current, when required, is mainly obtained from an a.c. supply by the use of rectifiers. Nevertheless, an understanding of d.c. generator is important because it represents a logical introduction to the behavior of d.c. motors. Indeed many d.c. motors in industry actually operate as d.c. generators for a brief period.

Generator Principle

An electric generator is a machine that converts mechanical energy into electrical energy. An electric generator is based on the principle that whenever flux is cut by a conductor, an e.m.f. is induced which will cause a current to flow if the conductor circuit is closed. The direction of induced e.m.f. (and hence current) is given by Fleming's right hand rule. Therefore, the essential components of a generator are:

(a) a magnetic field
(b) conductor or a group of conductors
(c) motion of conductor w.r.t. magnetic field.

Simple Loop Generator

Consider a single turn loop ABCD rotating clockwise in a uniform magnetic field with a constant speed as shown in Fig.(1.1). As the loop rotates, the flux linking the coil sides AB and CD changes continuously. Hence the e.m.f. induced in these coil sides also changes but the e.m.f. induced in one coil side adds to that induced in the other.

(i) When the loop is in position no. 1 [See Fig. 1.1], the generated e.m.f. is zero because the coil sides (AB and CD) are cutting no flux but are moving parallel to it

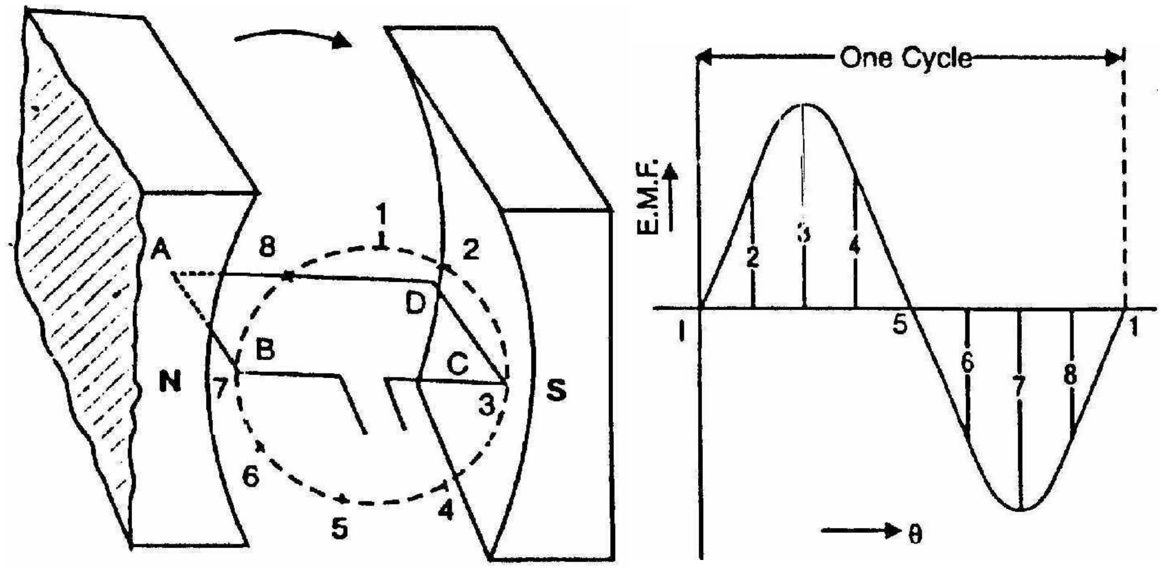
(ii) When the loop is in position no. 2, the coil sides are moving at an angle to the flux and, therefore, a low e.m.f. is generated as indicated by point 2 in Fig. (1.2).

(iii) When the loop is in position no. 3, the coil sides (AB and CD) are at right angle to the flux and are, therefore, cutting the flux at a maximum rate. Hence at this instant, the generated e.m.f. is maximum as indicated by point 3 in Fig. (1.2).

(iv) At position 4, the generated e.m.f. is less because the coil sides are cutting the flux at an angle.

(v) At position 5, no magnetic lines are cut and hence induced e.m.f. is zero as indicated by point 5 in Fig. (1.2).

(vi) At position 6, the coil sides move under a pole of opposite polarity and hence the direction of generated e.m.f. is reversed. The maximum e.m.f. in this direction (i.e., reverse direction, See Fig. 1.2) will be when the loop is at position 7 and zero when at position 1. This cycle repeats with each revolution of the coil.



Note that e.m.f. generated in the loop is alternating one. It is because any coilside, say AB has e.m.f. in one direction when under the influence of N-pole and in the other direction when under the influence of S-pole. If a load is connected across the ends of the loop, then alternating current will flow through the load.The alternating voltage generated in the loop can be converted into direct voltage by a device called commutator. We then have the d.c. generator. In fact, a commutator is a mechanical rectifier.

E.M.F. Equation of a D.C. Generator

Let ϕ = flux/pole in Wb

Z = total number of armature conductors

P = number of poles

A = number of parallel paths = 2 ... for wave winding

= P ... for lap winding

N = speed of armature in r.p.m.

E_g = e.m.f. of the generator = e.m.f./parallel path

Flux cut by one conductor in one revolution of the armature,

$$d\phi = P\phi \text{ webers}$$

Time taken to complete one revolution,

$$dt = 60/N \text{ second}$$

$$\text{e.m.f generated/conductor} = \frac{d\phi}{dt} = \frac{P\phi}{60/N} = \frac{P\phi N}{60} \text{ volts}$$

e.m.f. of generator,

E_g = e.m.f. per parallel path

= (e.m.f/conductor) × No. of conductors in series per parallel path

$$= \frac{P\phi N}{60} \times \frac{Z}{A}$$

$$\therefore \quad E_g = \frac{P\phi ZN}{60 A}$$

where $A = 2$ for-wave winding

$A = P$ for lap winding

Types of D.C. Generators

The magnetic field in a d.c. generator is normally produced by electromagnets rather than permanent magnets. Generators are generally classified according to their methods of field excitation. On this basis, d.c. generators are divided into the following two classes:

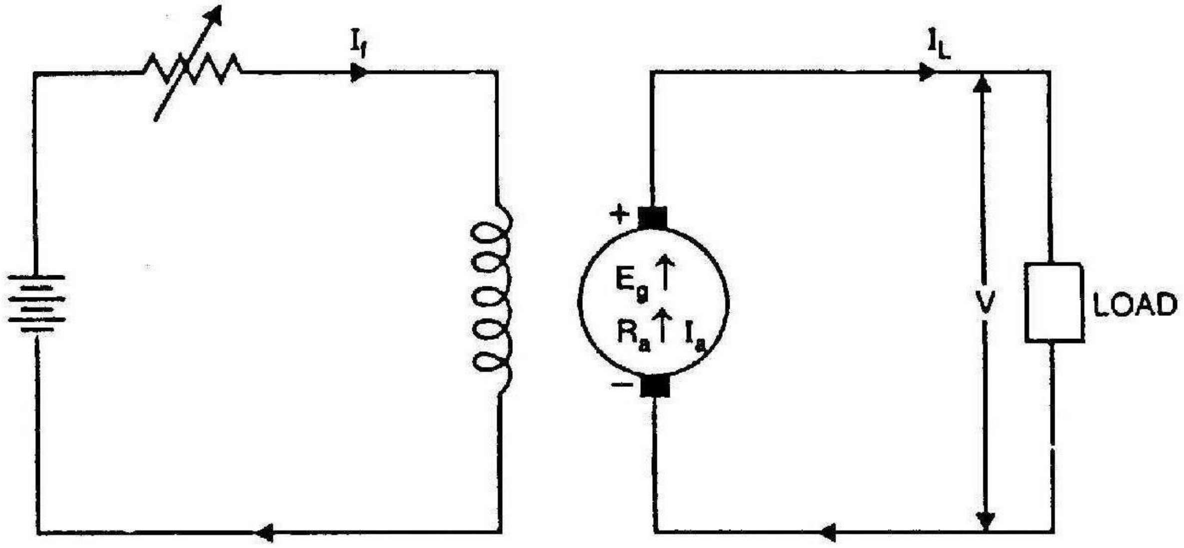
(i) Separately excited d.c. generators

(ii) Self-excited d.c. generators

The behaviour of a d.c. generator on load depends upon the method of field excitation adopted

Separately Excited D.C. Generators

A d.c. generator whose field magnet winding is supplied from an independent external d.c. source (e.g., a battery etc.) is called a separately excited generator. Fig. (1.32) shows the connections of a separately excited generator. The voltage output depends upon the speed of rotation of armature and the field current (Eg = P ZN/60 A). The greater the speed and field current, greater is the generatede.m.f. It may be noted that separately excited d.c. generators are rarely used in practice. The d.c. generators are normally of self-excited type.



Armature current, $I_a = I_L$

Terminal voltage, $V = E_g - I_a R_a$

Electric power developed = $E_g I_a$

Power delivered to load = $E_g I_a - I_a^2 R_a = I_a (E_g - I_a R_a) = V I_a$

Self-Excited D.C. Generators

A d.c. generator whose field magnet winding is supplied current from the output of the generator itself is called a self-excited generator. There are three types of self-excited generators depending upon the manner in which the field winding is connected to the armature, namely;

(i) Series generator;

(ii) Shunt generator;

(iii) Compound generator

(i) Series generator

In a series wound generator, the field winding is connected in series with armature winding so that whole armature current flows through the field winding as well as the load. Fig. (1.33) shows the connections of a series wound generator. Since the field winding carries the whole of load current, it has a few turns of thick wire having low resistance. Series generators are rarely used except for special purposes e.g., as boosters.

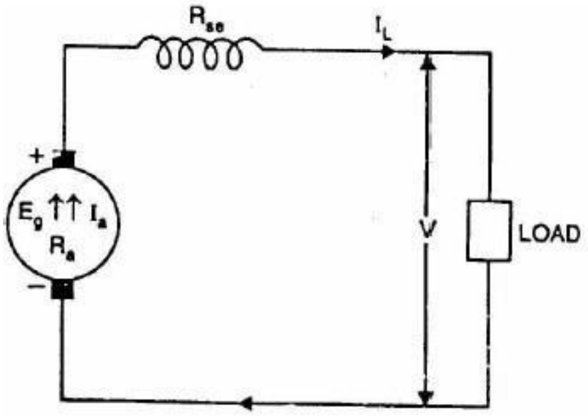
Armature current, $I_a = I_{se} = I_L = I(\text{say})$

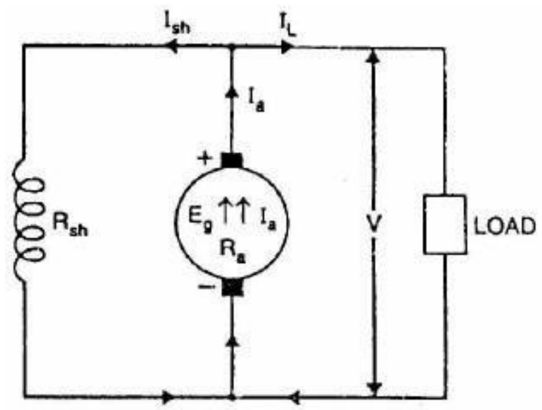
Terminal voltage, $V = E_G - I(R_a + R_{se})$

Power developed in armature $= E_g I_a$

Power delivered to load

$$= E_g I_a - I_a^2(R_a + R_{se}) = I_a[E_g - I_a(R_a - R_{se})] = VI_a \text{ or } VI_L$$





(ii) Shunt generator

In a shunt generator, the field winding is connected in parallel with the armature winding so that terminal voltage of the generator is applied across it. The shunt field winding has many turns of fine wire having high resistance. Therefore, only a part of armature current flows through shunt field winding and the rest flows through the load. Fig. (1.34) shows the connections of a shunt-wound generator.

Shunt field current, $I_{sh} = V/R_{sh}$

Armature current, $I_a = I_L + I_{sh}$

Terminal voltage, $V = E_g - I_a R_a$

Power developed in armature $= E_g I_a$

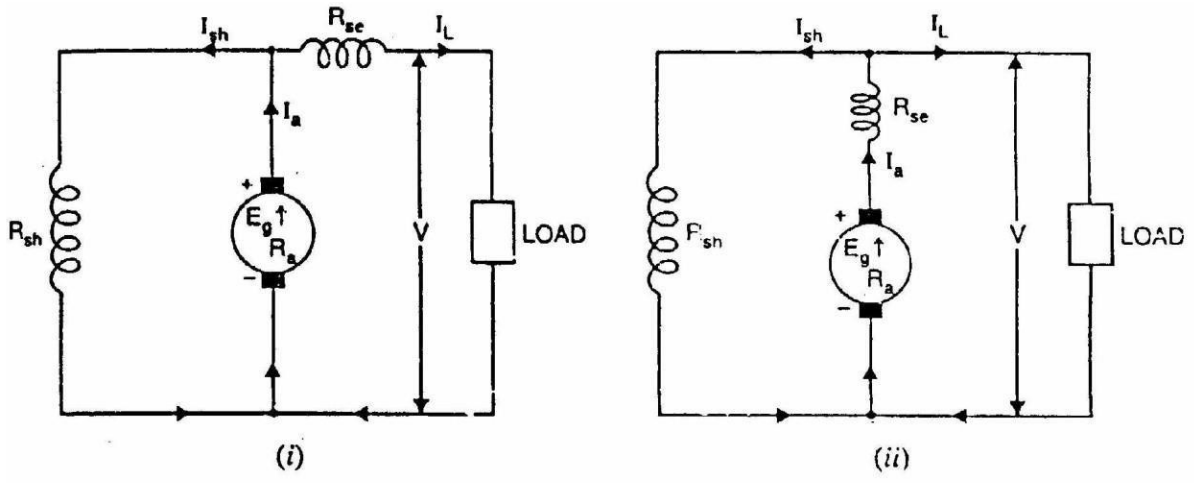
Power delivered to load $= VI_L$

(iii) Compound generator

In a compound-wound generator, there are two sets of field windings on each pole one is in series and the other in parallel with the armature. A compound wound generator may be:

(a) Short Shunt in which only shunt field winding is in parallel with the armature winding [See Fig. (i)].

(b) Long Shunt in which shunt field winding is in parallel with both series field and armature winding [See Fig. (ii)].



Short shunt

Series field current, $I_{se} = I_L$

Shunt field current, $I_{sh} = \dfrac{V + I_{se}R_{se}}{R_{sh}}$

Terminal voltage, $V = E_g - I_aR_a - I_{se}R_{se}$

Power developed in armature = E_gI_a

Power delivered to load = VI_L

Long shunt

Series field current, $I_{se} = I_a = I_L + I_{sh}$

Shunt field current, $I_{sh} = V/R_{sh}$

Terminal voltage, $V = E_g - I_a(R_a + R_{se})$

Power developed in armature = E_gI_a

Power delivered to load = VI_L

Q A 4-pole d.c. shunt generator has an armature resistance of 0.018 Ω. The armature is lap-wound with 520 conductors. When driven at 750 rev/min the machine produces a total armature current of 400 A at a terminal voltage of 200 V. Calculate the useful flux/pole.

A

$p = 2;\ a = 4;\ R_a = 0.018\,\Omega;\ z = 520;\ n = \frac{750}{60} = 12.5$ rev/s

$I_a = 400$ A; $V = 200$ V

$E = V + I_a R_a$ volt $= 200 + (400 \times 0.018)$

$E = 207.2$ V

$E = \frac{2p\Phi zn}{a}$ volt

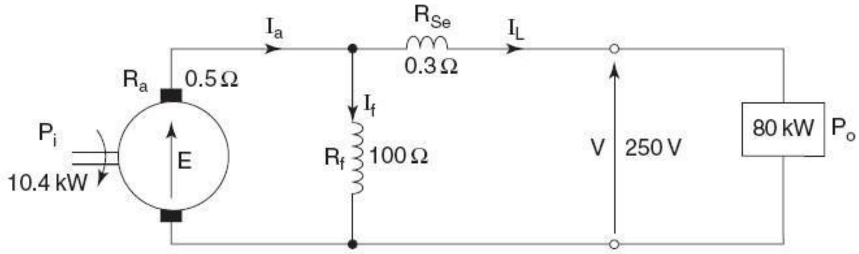
so, $\Phi = \frac{Ea}{2pzn}$ weber $= \frac{207.2 \times 4}{4 \times 520 \times 12.5}$

$\Phi = 31.9$ mWb **Ans**

Q **A short-shunt compound generator has armature, shunt field and series field resistances of 0.5 Ω 100 Ω, and 0.3 Ω respectively. When supplying a load of 8 kW at a terminal voltage of 250 V the input power supplied by the driving motor is 10.4 kW. Calculate (a) the generated emf, (b) the efficiency, (c) the iron, friction and windage loss, and (d) the total fixed losses.**

A

$R_a = 0.5\,\Omega;\ R_f = 100\,\Omega;\ R_{Se} = 0.3\,\Omega;\ P_o = 8000$ W; $V = 250$ V; $P_i = 10\,400$ W



(a) $I_L = \frac{P_O}{V_L}$ amp $= \frac{8000}{250} = 32$ A

$V_f = V + I_L R_{Se}$ volt $= 250 + (32 \times 0.3)$

$V_f = 259.6$ V

$I_f = \frac{V_f}{R_f}$ amp $= \frac{259.6}{100} = 2.6$ A

$I_a = I_L + I_f$ amp $= 32 + 2.6 = 34.6$ A

$E = V_f + I_a R_a$ volt $= 259.6 + (34.6 \times 0.5)$

$E = 276.9$ V **Ans**

(b) $\eta = \frac{P_o}{P_i} \times 100\% = \frac{8}{10.4} \times 100\%$

$\eta = 76.92\%$ **Ans**

(c) From the power flow diagram

$P_{Fe} = P_i - EI_a$ watt $= 10\,400 - (276.9 \times 34.6)$

$P_{Fe} = 819.26$ W **Ans**

(d) total fixed losses $= P_{Fe} + I_f^2 R_f$ watt

$= 819.26 + (2.6 \times 100)$

total fixed losses $= 1.08$ kW **Ans**

D.C. Motors

Introduction

D. C. motors are seldom used in ordinary applications because all electric supply companies furnish alternating current However, for special applications such as in steel mills, mines and electric trains, it is advantageous to convert alternating current into direct current in order to use d.c. motors. The reason is that speed/torque characteristics of d.c. motors are much more superior to that of a.c. motors. Therefore, it is not surprising to note that for industrial drives, d.c. motors are as popular as 3-phase induction motors. Like d.c. generators, d.c. motors are also of three types viz., series-wound, shunt-wound and compound wound. The use of a particular motor depends upon the mechanical load it has to drive.

D.C. Motor Principle

A machine that converts d.c. power into mechanical power is known as a d.c. motor. Its operation is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force. The direction of this force is given by Fleming's left hand rule and magnitude is given by; FBInewtons

Basically, there is no constructional difference between a d.c. motor and a d.c. generator. The same d.c. machine can be run as a generator or motor

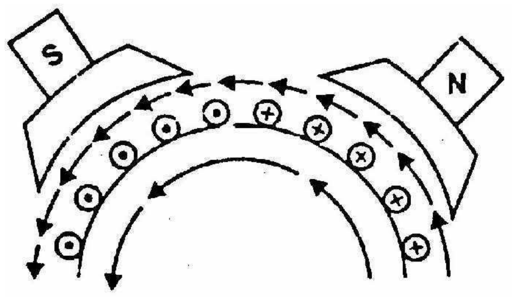
Working of D.C. Motor

Consider a part of a multipolar d.c. motor as shown in Fig. (4.1). When the terminals of the motor are connected to an external source of d.c. supply:

(i) the field magnets are excited developing alternate N and S poles

(ii) the armature conductors carry ^currents. All conductors under N-pole carry currents in one direction while all the conductors under S-pole carry currents in the opposite direction.

Suppose the conductors under N-pole carry currents into the plane of the paper and those under S-pole carry currents out of the plane of the paper as shown in Fig.(4.1). Since each armature conductor is carrying current and is placed in the magnetic field, mechanical force acts on it.



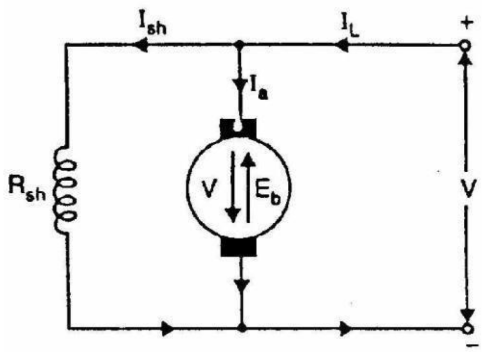
Referring to Fig. (4.1) and applying Fleming's left hand rule, it is clear that force on each conductor is tending to rotate the armature in anticlockwise direction. All these forces add together to produce a driving torque which sets the armature rotating.

When the conductor moves from one side of a brush to the other, the current in that conductor is reversed and at the same time it comes under the influence of next pole which is of opposite polarity.

Consequently, the direction of force on the conductor remains the same.

Back or Counter E.M.F.

When the armature of a d.c. motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence e.m.f. is induced in them as in a generator The induced e.m.f. acts in opposite direction to the applied voltage V(Lenz's law) and in known as back or counter e.m.f. Eb. The back e.m.f. Eb(= P□□ZN/60 A) is always less than the applied voltage V, although this difference is small when the motor is running under normal conditions.



Consider a shunt wound motor shown in Fig. (4.2). When d.c. voltage V is applied across the motor terminals, the field magnets are excited and armature conductors are supplied with current. Therefore, driving torque acts on the armature which begins to rotate. As the armature rotates, back e.m.f. Eb is induced which opposes the applied voltage V. The applied voltage V has to force current through the armature against the back e.m.f. Eb. The electric work done in overcoming and causing the current to flow against Eb is converted into mechanical energy developed in the armature. It follows, therefore, that energy conversion in a d.c. motor is only possible due to the production of back e.m.f. Eb.

Net voltage across armature circuit = $V - E_b$

If R_a is the armature circuit resistance, then, $I_a = \dfrac{V - E_b}{R_a}$

Since V and Ra are usually fixed, the value of Eb will determine the current drawn by the motor. If the speed of the motor is high, then back e.m.f. Eb (= P□ZN/60 A) is large and hence the motor will draw less armature current and viceversa.

Significance of Back E.M.F.

The presence of back e.m.f. makes the d.c. motor a self-regulating machine i.e., it makes the motor to draw as much armature current as is just sufficient to develop the torque required by the load.

$$\text{Armature current, } I_a = \frac{V - E_b}{R_a}$$

(i) When the motor is running on no load, small torque is required to overcome the friction and windage losses. Therefore, the armature current Ia is small and the back e.m.f. is nearly equal to the applied voltage.

(ii) If the motor is suddenly loaded, the first effect is to cause the armature to slow down. Therefore, the speed at which the armature conductors move through the field is reduced and hence the back e.m.f. Eb falls. The decreased back e.m.f. allows a larger current to flow through the armature and larger current means increased driving torque. Thus, the driving torque increases as the motor slows down. The motor will stop slowing down when the armature current is just sufficient to produce the increased torque required by the load.

(iii) If the load on the motor is decreased, the driving torque is momentarily in excess of the requirement so that armature is accelerated. As the armature speed increases, the back e.m.f. Eb also increases and causes the armature current Ia to decrease. The motor will stop accelerating when the armature current is just sufficient to produce the reduced torque required by the load.

It follows, therefore, that back e.m.f. in a d.c. motor regulates the flow of armature current i.e., it automatically changes the armature current to meet the load requirement.

Armature Torque of D.C. Motor

Torque is the turning moment of a force about an axis and is measured by the product of force (F) and radius (r) at right angle to which the force acts i.e. D.C. Motors

$$T = F \times r$$

In a d.c. motor, each conductor is acted upon by a circumferential force F at a distance r, the radius of the armature (Fig. 4.8). Therefore, each conductor exerts a torque, tending to rotate the armature. The sum of the torques due to all armature conductors is known as gross or armature torque (Ta).

Torque due to one conductor = F × r newton- metre

Total armature torque, $T_a = Z\,F\,r$ newton-metre

$= Z\,B\,i\,\ell\,r$

Now $i = I_a/A$, $B = \phi/a$ where a is the x-sectional area of flux path per pole at radius r. Clearly, $a = 2\pi r\ell/P$.

$$\therefore \quad T_a = Z \times \left(\frac{\phi}{2}\right) \times \left(\frac{I_a}{A}\right) \times \ell \times r$$

$$= Z \times \frac{\phi}{2\pi r\ell/P} \times \frac{I_a}{A} \times \ell \times r = \frac{Z\phi I_a P}{2\pi A}\ \text{N - m}$$

or $$T_a = 0.159\, Z\phi I_a \left(\frac{P}{A}\right)\ \text{N - m} \qquad \text{(i)}$$

Since Z, P and A are fixed for a given machine,

$\therefore \quad T_a \propto \phi I_a$

Hence torque in a d.c. motor is directly proportional to flux per pole and armature current.

(i) For a shunt motor, flux ϕ is practically constant.

$\therefore \quad T_a \propto I_a$

(ii) For a series motor, flux ϕ is directly proportional to armature current I_a provided magnetic saturation does not take place.

$\therefore \quad T_a \propto I_a^2$

Necessity of D.C. Motor Starter

At starting, when the motor is stationary, there is no back e.m.f. in the armature. Consequently, if the motor is directly switched on to the mains, the armature will draw a heavy current ($Ia = V/Ra$) because of small armature resistance.

As an example, 5 H.P., 220 V shunt motor has a full-load current of 20 A and an armature resistance of about 0.5 Ω. If this motor is directly switched on to supply, it would take an armature current of 220/0.5 = 440 A which is 22 times the full-load current. This high starting current may result in:

(i) burning of armature due to excessive heating effect,

(ii) damaging the commutator and brushes due to heavy sparking,

(iii) excessive voltage drop in the line to which the motor is connected. The result is that the operation of other appliances connected to the line may be impaired and in particular cases, they may refuse to work. In order to avoid excessive current at starting, a variable resistance (known as starting resistance) is inserted in series with the armature circuit. This resistance is gradually reduced as the motor gains speed (and hence Eb increases) and eventually it is cut out completely when the motor has attained full speed. The value of starting resistance is generally such that starting current is limited to 1.25 to 2 times the full-load current.

Types of D.C. Motor Starters

The stalling operation of a d.c. motor consists in the insertion of external resistance into the armature circuit to limit the starting current taken by the motor and the removal of this resistance in steps as the motor accelerates. When the motor attains the normal speed, this resistance is totally cut out of the armature circuit. It is very important and desirable to provide the starter with protective devices to enable the starter arm to return to OFF position

(i) when the supply fails, thus preventing the armature being directly across the mains when this voltage is restored. For this purpose, we use no-volt release coil.

(ii) when the motor becomes overloaded or develops a fault causing the motor to take an excessive current. For this purpose, we use overload release coil. There are two principal types of d.c. motor starters viz., three-point starter and four-point starter. As we shall see, the two types of starters differ only in the manner in which the no-volt release coil is connected.

Three-Point Starter

This type of starter is widely used for starting shunt and compound motors.

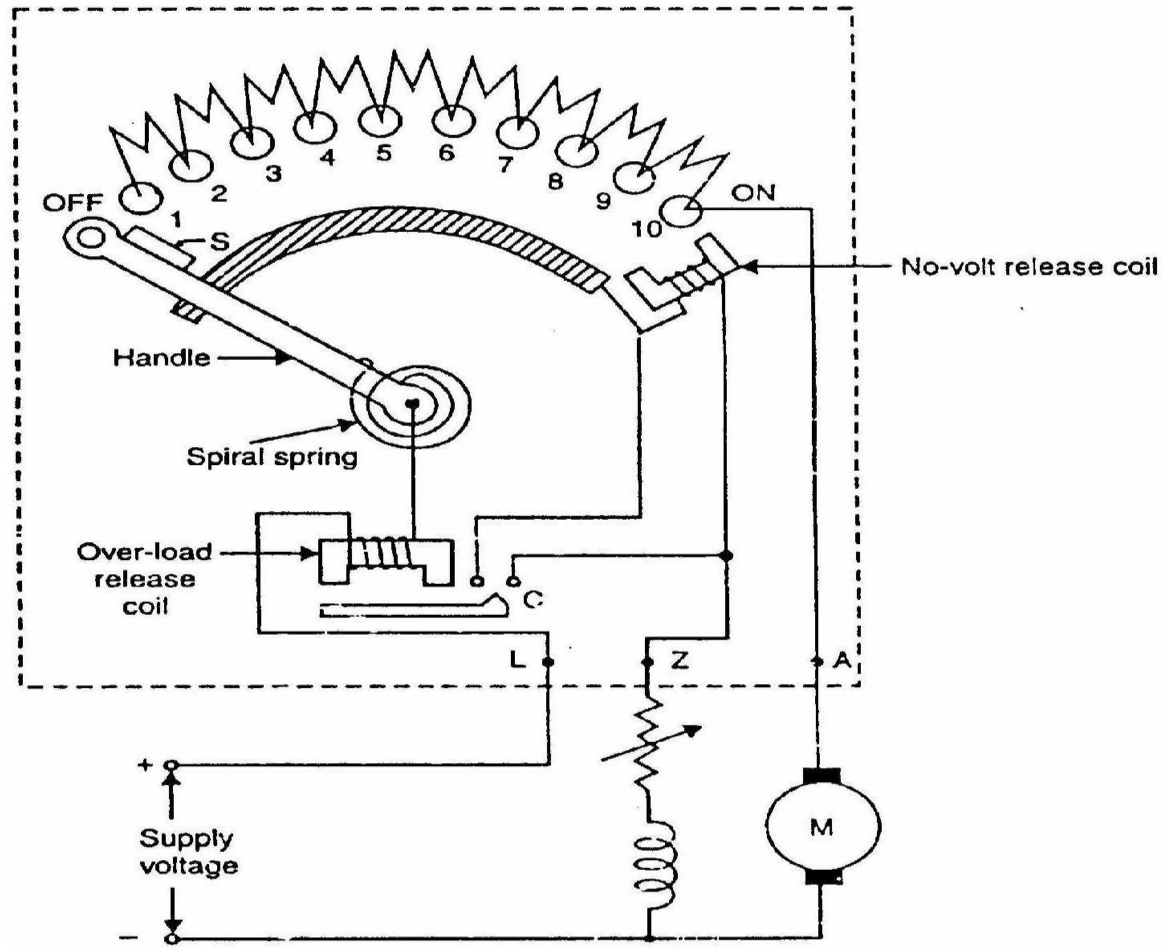
Schematic diagram

Fig. (5.16) shows the schematic diagram of a three-point starter for a shunt motor with protective devices. It is so called because it has three terminals L, Z and A. The starter consists of starting resistance divided into several sections and connected in series with the armature. The tapping points of the starting resistance are brought out to a number of studs. The three terminals L, Z and A of the starter are connected respectively to the positive line terminal, shunt field terminal and armature terminal. The other terminals of the armature and shunt field windings are connected to the negative terminal of the supply. The no-volt release coil is connected in the shunt field circuit. One end of the handle is connected to the terminal L through the over-load release coil. The other end of the handle moves against a spiral spring and makes contact with each stud during starting operation, cutting out more and more starting resistance as it passes over each stud in clockwise direction.

Operation

(i) To start with, the d.c. supply is switched on with handle in the OFF position.

(ii) The handle is now moved clockwise to the first stud. As soon as it comes in contact with the first stud, the shunt field winding is directly connected across the supply, while the whole starting resistance is inserted in series with the armature circuit.
(iii) As the handle is gradually moved over to the final stud, the starting resistance is cut out of the armature circuit in steps. The handle is now held magnetically by the no-volt release coil which is energized by shunt field current.
(iv) If the supply voltage is suddenly interrupted or if the field excitation is accidentally cut, the no-volt release coil is demagnetized and the handle goes back to the OFF position under the pull of the spring. If no-volt release coil were not used, then in case of failure of supply, the handle would remain on the final stud. If then supply is restored, the motor will be directly connected across the supply, resulting in an excessive armature current.
(v) If the motor is over-loaded (or a fault occurs), it will draw excessive current from the supply. This current will increase the ampere-turns of the over-load release coil and pull the armature C, thus short-circuiting the no-volt release coil. The no-volt coil is demagnetized and the handle is pulled to the OFF position by the spring. Thus, the motor is automatically disconnected from the supply.



Drawback

In a three-point starter, the no-volt release coil is connected in series with the shunt field circuit so that it carries the shunt field current. While exercising speed control through field regulator, the field current may be weakened to such an extent that the no-volt release coil may not be able to keep the starter arm in the ON position. This may disconnect the motor from the supply when it is not desired. This drawback is overcome in the four point starter.

Speed Control of D.C. Motors

Introduction

Although a far greater percentage of electric motors in service are a.c. motors, the d.c. motor is of considerable industrial importance. The principal advantage of a d.c. motor is that its speed can be changed over a wide range by a variety of simple methods. Such a fine speed control is generally not possible with a.c. motors. In fact, fine speed control is one of the reasons for the strong competitive position of d.c. motors in the modem industrial applications.

the various methods of-speed control of d.c. motors are:

Speed Control of D.C. Motors

The speed of a d.c. motor is given by:

$$N \propto \frac{E_b}{\phi}$$

$$N = K\frac{(V - I_a R)}{\phi} \text{ r.p.m.}$$

$$R = R_a \qquad \text{for shunt motor}$$

$$= R_a + R_{se}. \qquad \text{for series motor}$$

it is clear that there are three main methods of controlling the speed of a d.c. motor, namely:

(i) By varying the flux per pole (ϕ). This is known as flux control method.

(ii) By varying the resistance in the armature circuit. This is known as armature control method.

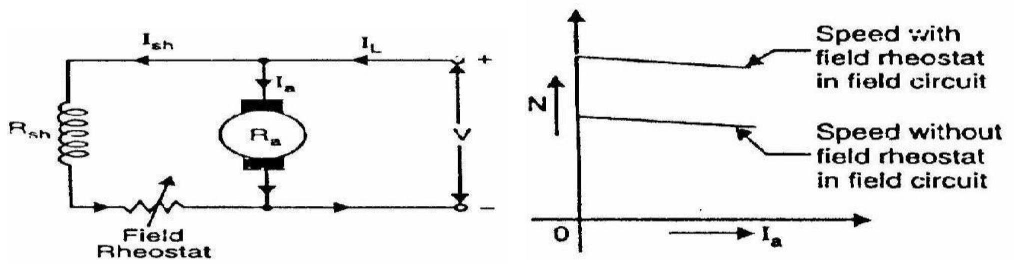
(iii) By varying the applied voltage V. This is known as voltage control method.

Speed Control of D.C. Shunt Motors

The speed of a shunt motor can be changed by (i) flux control method
(ii) armature control method (iii) voltage control method.
The first method (i.e. flux control method) is frequently used because it is simple and inexpensive.

1. Flux control method

It is based on the fact that by varying the flux ϕ, the motor speed ($N \propto 1/\phi$) canbe changed and hence the name flux control method. In this method, a variable resistance (known as shunt field rheostat) is placed in series with shunt field winding as shown in Fig.



The shunt field rheostat reduces the shunt field current Ish and hence the flux. Therefore, we can only raise the speed of the motor above the normal speed (See Fig. 5.2). Generally, this method permits to increase the speed in the ratio 3:1. Wider speed ranges tend to produce instability and poor commutation.

Advantages

(i) This is an easy and convenient method.

(ii) It is an inexpensive method since very little power is wasted in the shunt field rheostat due to relatively small value of Ish.

(iii) The speed control exercised by this method is independent of load on the machine.

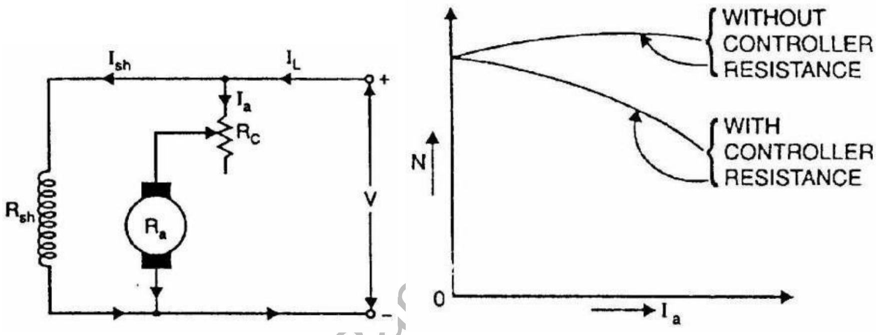
Disadvantages

(i) Only speeds higher than the normal speed can be obtained since the total field circuit resistance cannot be reduced below Rsh—the shunt field winding resistance.

(ii) There is a limit to the maximum speed obtainable by this method. It is because if the flux is too much weakened, commutation becomes poorer.

2. Armature control method

This method is based on the fact that by varying the voltage available across the armature, the back e.m.f and hence the speed of the motor can be changed. This is done by inserting a variable resistance RC (known as controller resistance) in series with the armature as shown in Fig



$$N \propto V - I_a(R_a + R_C)$$

R_C = controller resistance

Due to voltage drop in the controller resistance, the back e.m.f. (Eb) is decreased. Since $N \propto E_b$, the speed of the motor is reduced. The highest speed obtainable is lhat corresponding to RC = 0 i.e., normal speed. Hence, this method can only provide speeds below the normal speed

Disadvantages

(i) A large amount of power is wasted in the controller resistance since it carries full armature current Ia.

(ii) The speed varies widely with load since the speed depends upon the voltage drop in the controller resistance and hence on the armature current demanded by the load.

(iii) The output and efficiency of the motor are reduced.

(iv) This method results in poor speed regulation.

Due to above disadvantages, this method is seldom used to control tie speed of shunt motors.

Note. The armature control method is a very common method for the speed control of d.c. series motors. The disadvantage of poor speed regulation is not important in a series motor which is used only where varying speed service is required.

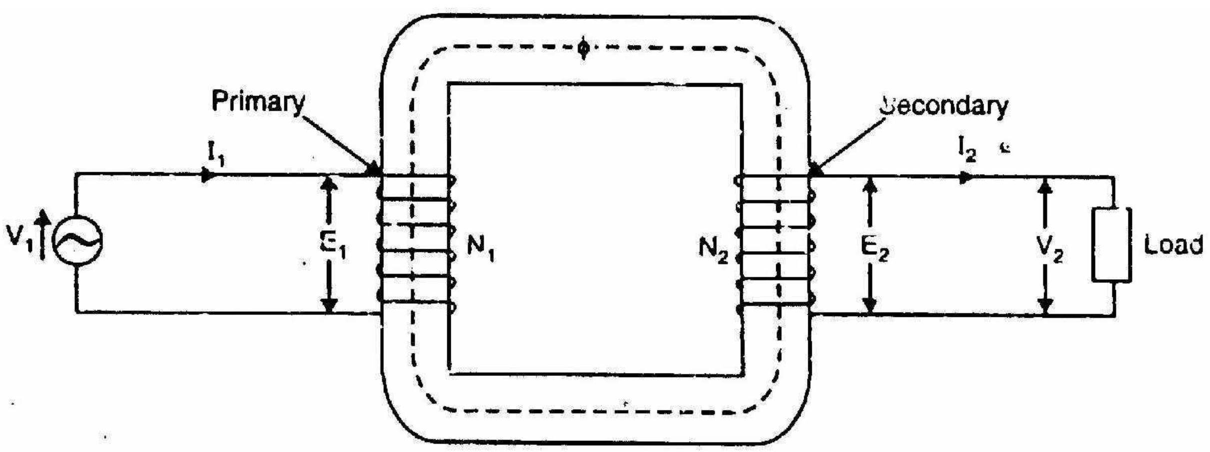
3. Transformer

Introduction

The transformer is probably one of the most useful electrical devices ever invented. It can change the magnitude of alternating voltage or current from one value to another. This useful property of transformer is mainly responsible for the widespread use of alternating currents rather than direct currents i.e., electric power is generated, transmitted and distributed in the form of alternating current. Transformers have no moving parts, rugged and durable in construction, thus requiring very little attention. They also have a very high efficiency—as high as 99%. In this chapter, we shall study some of the basic properties of transformers.

Transformer

A transformer is a static piece of equipment used either for raising or lowering the voltage of an a.c. supply with a corresponding decrease or increase in current. It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in Fig. . The winding connected to the a.c. source is called primary winding (or primary) and the one connected to load is called secondary winding (or secondary). The alternating voltage V1 whose magnitude is to be changed is applied to the primary. Depending upon the number of turns of the primary (N1) and secondary (N2), an alternating e.m.f. E2 is induced in the secondary. This induced e.m.f. E2 in the secondary causes a secondary current I2. Consequently, terminal voltage V2 will appear across the load. If V2 > V1, it is called a step up-transformer. On the other hand, if V2 < V1, it is called a step-down transformer.



Working

When an alternating voltage V1 is applied to the primary, an alternating flux is set up in the core. This alternating flux links both the windings and induces e.m.f.s E1 and E2 in them according to Faraday's laws of electromagnetic induction. The e.m.f. E1 is termed as primary e.m.f. and e.m.f. E2 is termed as secondary e.m.f.

$$E_1 = -N_1 \frac{d\phi}{dt}$$

$$E_2 = -N_2 \frac{d\phi}{dt}$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Note that magnitudes of E2 and E1 depend upon the number of turns on the secondary and primary respectively. If N2 > N1, then E2 > E1 (or V2 > V1) and we get a step-up transformer. On the other hand, if N2 < N1, then E2 < E1 (or V2 < V1) and we get a step-down transformer. If load is connected across the secondary winding, the secondary e.m.f. E2 will cause a current I2 to flow through the load. Thus, a transformer enables us to transfer a.c. power from one circuit to another with a change in voltage level. The following points may be noted carefully:

(i) The transformer action is based on the laws of electromagnetic induction.

(ii) There is no electrical connection between the primary and secondary.

The a.c. power is transferred from primary to secondary through magnetic flux.

(iii) There is no change in frequency i.e., output power has the same frequency as the input power.

(iv) The losses that occur in a transformer are:

(a) core losses—eddy current and hysteresis losses

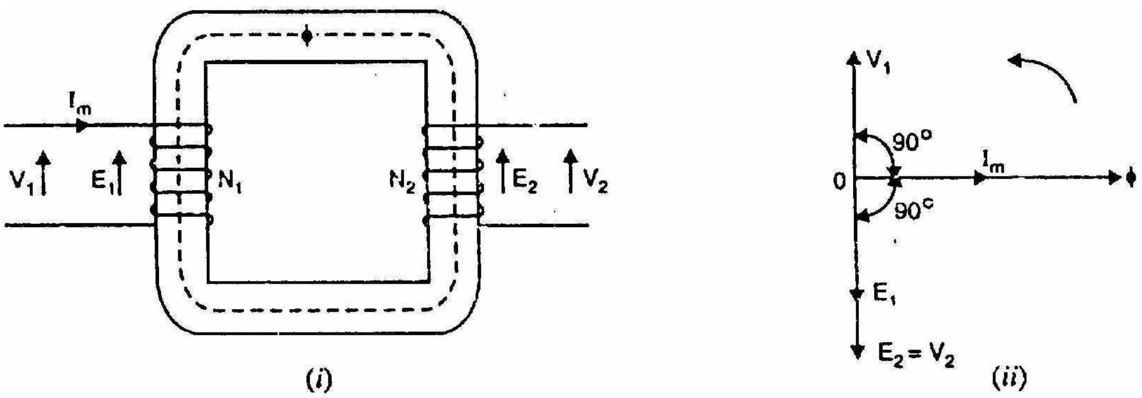
(b) copper losses—in the resistance of the windings

In practice, these losses are very small so that output power is nearly equal to the input primary power. In other words, a transformer has very high efficiency.

E.M.F. Equation of a Transformer

Consider that an alternating voltage V1 of frequency f is applied to the primary as shown in Fig. (i)). The sinusoidal flux produced by the primary can be represented as:

The instantaneous e.m.f. e1 induced in the primary is



$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\phi_m \sin \omega t)$$
$$= -\omega N_1 \phi_m \cos \omega t = -2\pi f N_1 \phi_m \cos \omega t$$
$$= 2\pi f N_1 \phi_m \sin(\omega t - 90°) \qquad \text{(i)}$$

It is clear from the above equation that maximum value of induced e.m.f. in the primary is

$$E_{m1} = 2\pi f N_1 \phi_m$$

The r.m.s. value E^ of the primary e.m.f. is

$$E_1 = \frac{E_{m1}}{\sqrt{2}} = \frac{2\pi f N_1 \phi_m}{\sqrt{2}}$$

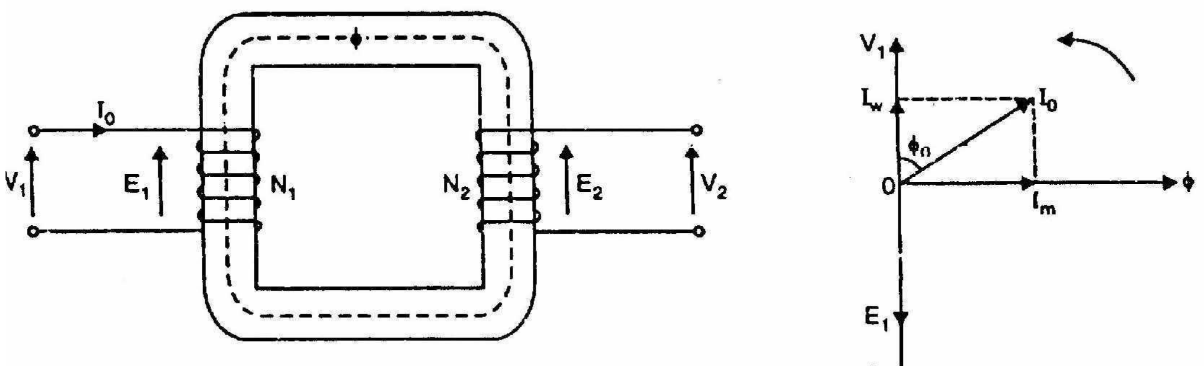
or $\quad E_1 = 4.44 f N_1 \phi_m$

Similarly $\quad E_2 = 4.44 f N_2 \phi_m$

In an ideal transformer, $E_1 = V_1$ and $E_2 = V_2$.

Phasor diagram.

Consider a practical transformer on no load i.e., secondary on open-circuit as shown in Fig. (i)). The primary will draw a small current I0 to supply (i) the iron losses and (ii) a very small amount of copper loss in the primary. Hence the primary no load current I0 is not 90° behind the applied voltage V1 but lags it by an angle $\phi_0 < 90°$ as shown in the phasor diagram in Fig. ((ii)). No load input power, W0 = V1 I0 cos ϕ_0

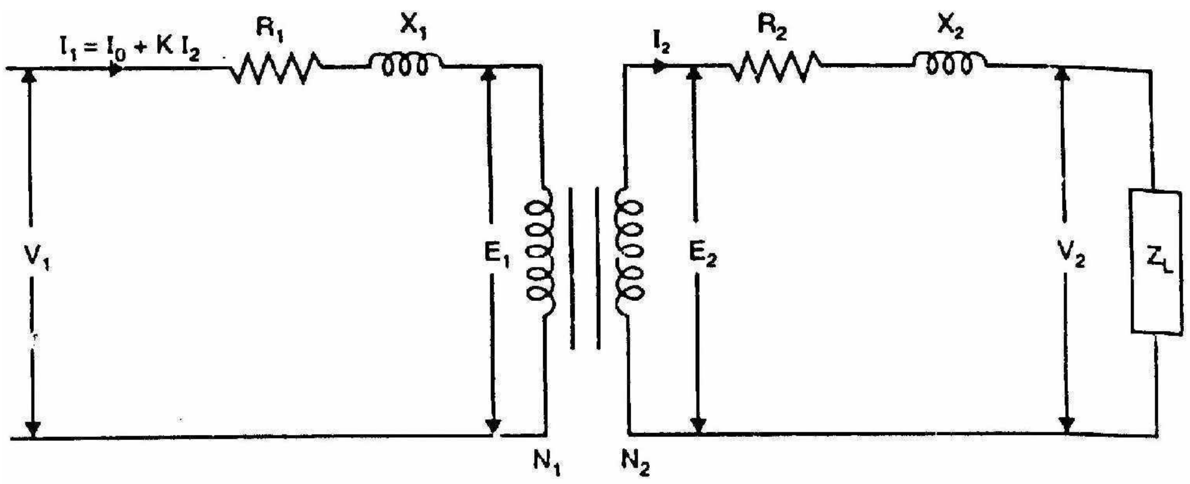


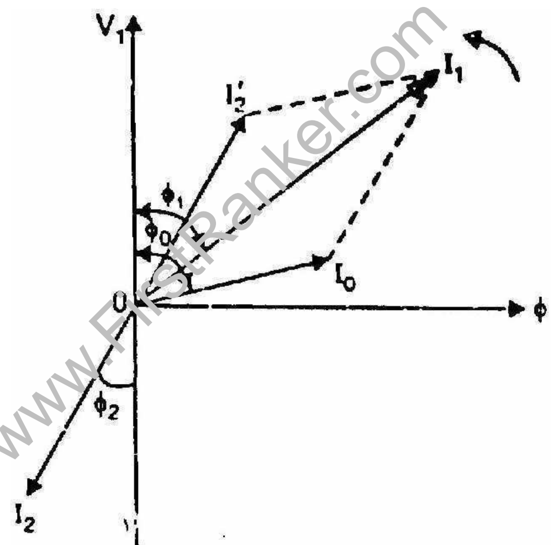
(i) (ii)

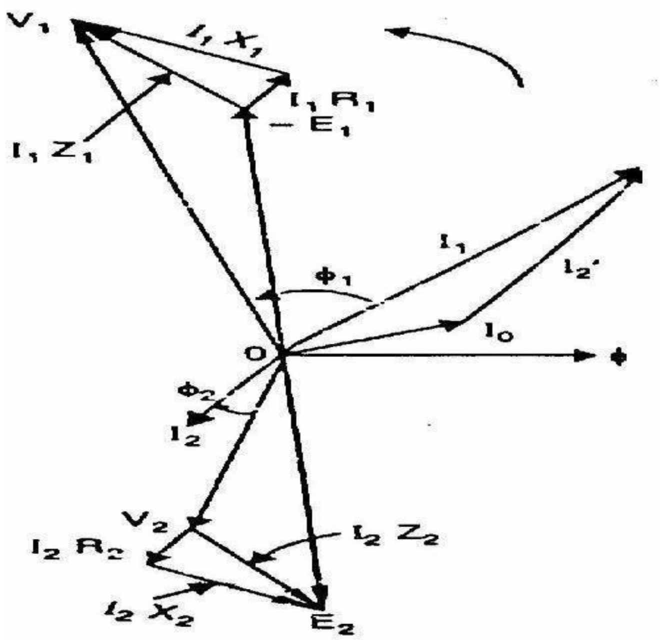
Fig. shows the phasor diagram for the usual case of inductive load.

Both E1 and E2 lag behind the mutual flux ϕ by 90°. The current I'2 represents the primary current to neutralize the demagnetizing effect of secondary current I2. Now I'2 = K I2 and is antiphase with I2. I0 is the no-load current of the transformer. The phasor sum of I'2 and

I0 gives the total primary current I1. Note that in drawing the phasor diagram, the value of K is assumed to be unity so that primary phasors are equal to secondary phasors.
Primary p.f. = cos □1
Secondary p.f. = cos □2
Primary input power = V1 I1 cos □1
Secondary output power = V1 I2 cos □2

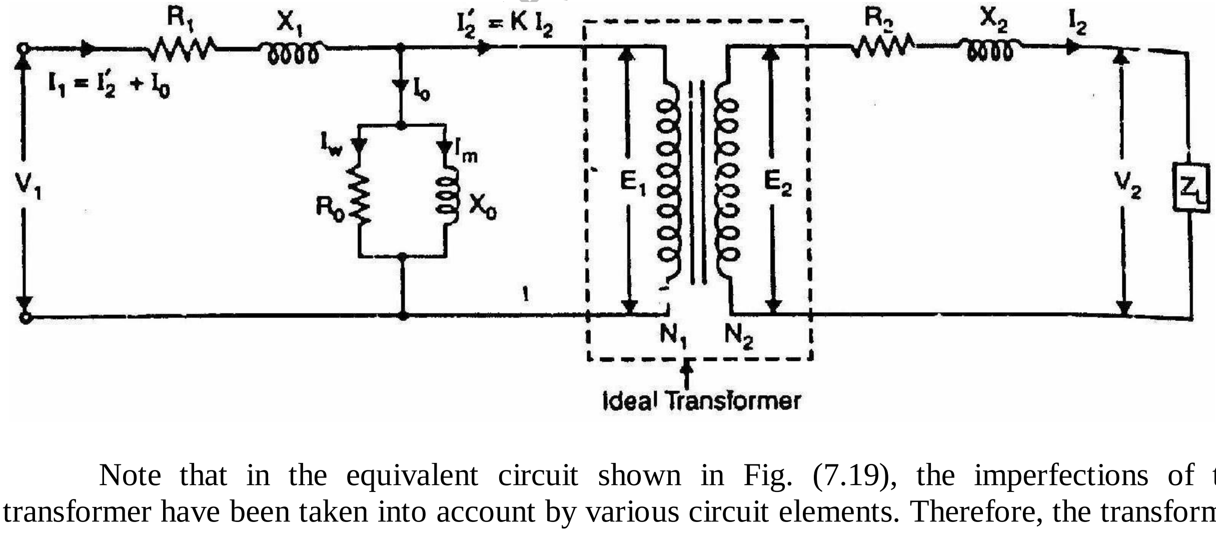






Equivalent Circuit of a Transformer

Fig shows the exact equivalent circuit of a transformer on load. Here R1 is the primary winding resistance and R2 is the secondary winding resistance. Similarly, X1 is the leakage reactance of primary winding and X2 is the leakage reactance of the secondary winding. The parallel circuit R0 X0 is the no-load equivalent circuit of the transformer. The resistance R0 represents the core losses (hysteresis and eddy current losses) so that current IW which supplies the core losses is shown passing through R0. The inductive reactance X0 represents a loss-free coil which passes the magnetizing current Im. The phasor sum of IW and Im is the no-load current I0 of the transformer.



Note that in the equivalent circuit shown in Fig. (7.19), the imperfections of the transformer have been taken into account by various circuit elements. Therefore, the transformer is now the ideal one. Note that equivalent circuit has created two normal electrical circuits separated only by an ideal transformer whose function is to change values according to the equation:

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I'_2}{I_2}$$

The following points may be noted from the equivalent circuit:

(i) When the transformer is on no-load (i.e., secondary terminals are opencircuited), there is no current in the secondary winding. However, the primary draws a small no-load current I0. The no-load primary current I0 is composed of (a) magnetizing current (Im) to create magnetic flux in the core and (b) the current IW required to supply the core losses.

(ii) When the secondary circuit of a transformer is closed through some external load ZL, the voltage E2 induced in the secondary by mutual flux will produce a secondary current I2. There will be I2 R2 and I2 X2 drops in the secondary winding so that load voltage V2 will be less than E2.

(iii) When the transformer is loaded to carry the secondary current I2, the primary current consists of two components:

(a) The no-load current I0 to provide magnetizing current and the current required to supply the core losses.

(b) The primary current I'2 (= K I2) required to supply the load connected to the secondary.

(iv) Since the transformer in Fig. in now ideal, the primary induced voltage E1 can be calculated from the relation:

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

If we add I_1R_1 and I_1X_1 drops to E_1, we get the primary input voltage V_1

$$V_1 = -E_1 + I_1(R_1 + j X_1) = -E_1 + I_1Z_1$$

or

$$V_1 = -E_1 + I_1Z_1$$

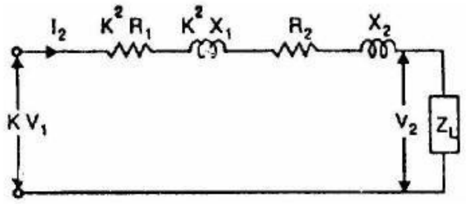
Voltage Drop in a Transformer

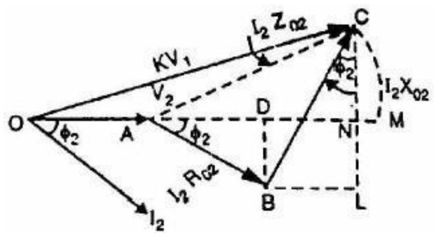
The approximate equivalent circuit of transformer referred to secondary is shown in Fig. At no-load, the secondary voltage is K V1. When a load having a lagging p.f. cos2 is applied, the secondary carries a current I2 and voltage drops occur in (R2 + K2 R1) and (X2 + K2 X1). Consequently, the secondary voltage falls from K V1 to V2. Referring to Fig. (7.28), we have,

$$\begin{aligned} V_2 &= KV_1 - I_2\left[\left(R_2 + K^2R_1\right) + j\left(X_2 + K^2X_1\right)\right] \\ &= KV_1 - I_2\left(R_{02} + j X_{02}\right) \\ &= KV_1 - V_2 = I_2Z_{02} \end{aligned}$$

Drop in secondary voltage $= KV_1 - V_2 = I_2 Z_{02}$

The phasor diagram is shown in Fig. (7.29). It is clear from the phasor diagram that drop in secondary voltage is AC = $I_2 Z_{02}$. It can be found as follows. With O as centre and OC as radius, draw an arc cutting OA produced at M. Then AC = AM = AN. From B, draw BD perpendicular to OA produced. Draw CN perpendicular to OM and draw BL || OM.





Approximate drop in secondary voltage

$$= AN = AD + DN$$

$$= AD + BL \qquad (\because \quad BL = DN)$$

$$= I_2R_{02} \cos\phi_2 + I_2X_{02} \sin\phi_2$$

For a load having a leading p.f. cos ϕ_2, we have,

$$\text{Approximate voltage drop } = I_2R_{02} \cos\phi_2 - I_2X_{02} \sin\phi_2$$

Note: If the circuit is referred to primary, then it can be easily established that:

$$\text{Approximate voltage drop } = I_1R_{01} \cos\phi_2 \pm I_1X_{01} \sin\phi_2$$

Voltage Regulation

The voltage regulation of a transformer is the arithmetic difference (not phasor difference) between the no-load secondary voltage (0V2) and the secondary voltage V2 on load expressed as percentage of no-load voltage i.e.

$$\%\text{age voltage regulation} = \frac{{}_0V_2 - V_2}{{}_0V_2} \times 100$$

${}_0V_2$ = No-load secondary voltage = K V_1
V_2 = Secondary voltage on load

$$ _0V_2 - V_2 = I_2R_{02} \cos\phi_2 \pm I_2X_{02} \sin\phi_2 $$

The +ve sign is for lagging p.f. and –ve sign for leading p.f.

It may be noted that %age voltage regulation of the transformer will be the same whether primary or secondary side is considered.

Transformer Tests

The circuit constants, efficiency and voltage regulation of a transformer can be determined by two simple tests (i) open-circuit test and (ii) short-circuit lest. These tests are very convenient as they provide the required information without actually loading the transformer. Further, the power required to carry out these tests is very small as compared with full-load output of the transformer. These tests consist of measuring the input voltage, current and power to the primary first with secondary open-circuited (open-circuit test) and then with the secondary short-circuited (short circuit test).

Open-Circuit or No-Load Test

This test is conducted to determine the iron losses (or core losses) and parameters R0 and X0 of the transformer. In this test, the rated voltage is applied to the primary (usually low-voltage winding) while the secondary is left opencircuited.

The applied primary voltage V1 is measured by the voltmeter, the noload current I0 by ammeter and no-load input power W0 by wattmeter as shown in Fig. (7.30 (i)). As the normal rated voltage is applied to the primary, therefore, normal iron losses will occur in the transformer core. Hence wattmeter will record the iron losses and small copper loss in the primary. Since no-load current I0 is very small (usually 2-10 % of rated current). Cu losses in the primary under no-load condition are negligible as compared with iron losses.

Hence, wattmeter reading practically gives the iron losses in the transformer. It is reminded that iron losses are the same at all loads. Fig. (7.30 (ii)) shows the equivalent circuit of transformer on no-load.

Iron losses, Pi = Wattmeter reading = W0

No load current = Ammeter reading = I0

Applied voltage = Voltmeter reading = V1

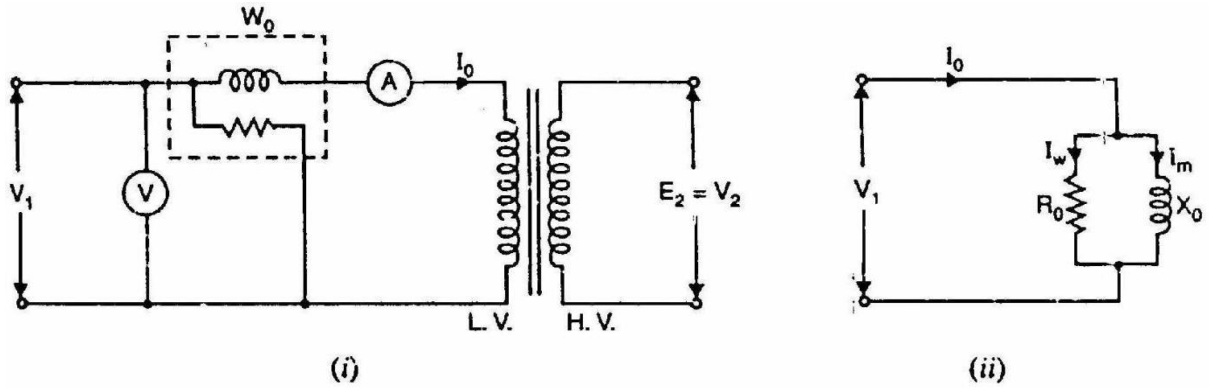
Inputpower,

$$ \therefore \quad \text{No-load p.f.,} \ \cos\phi_0 = \frac{W_0}{V_1} I_0 $$

$$ I_W = I_0 \cos\phi_0; \qquad I_m = I_0 \sin\phi_0 $$

$$ R_0 = \frac{V_1}{I_W} \quad \text{and} \quad X_0 = \frac{V_1}{I_m} $$

Thus open-circuit test enables us to determine iron losses and parameters R0 and X0 of the transformer.

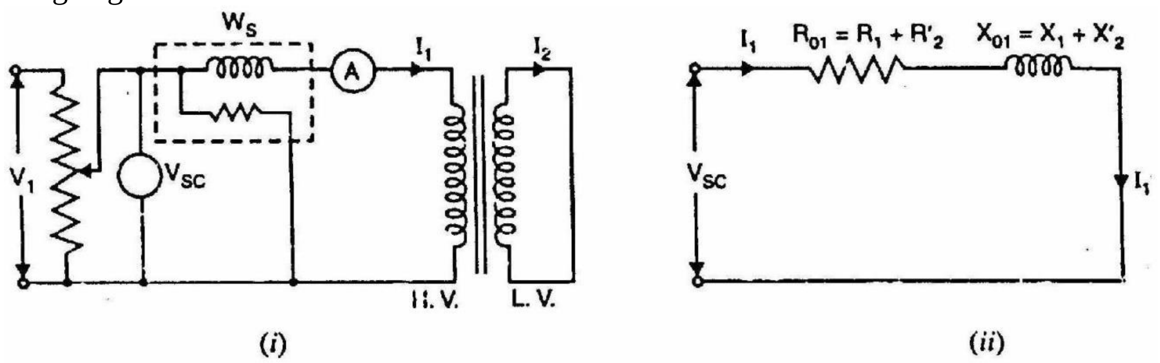


7.19 Short-Circuit or Impedance Test

This test is conducted to determine R01 (or R02), X01 (or X02) and full-load copper losses of the transformer. In this test, the secondary (usually low-voltage winding) is short-circuited by a thick conductor and variable low voltage is applied to the primary as shown in Fig. (i)). The low input voltage is gradually raised till at voltage VSC, full-load current I1 flows in the primary.

Then I2 in the secondary also has full-load value since I1/I2 = N2/N1. Under such conditions, the copper loss in the windings is the same as that on full load.

There is no output from the transformer under short-circuit conditions. Therefore, input power is all loss and this loss is almost entirely copper loss. It is because iron loss in the core is negligibly small since the voltage VSC is very small. Hence, the wattmeter will practically register the full-load copper losses in the transformer windings. Fig. (7.31 (ii)) shows the equivalent circuit of a transformer on short circuit as referred to primary; the no-load current being neglected due to its smallness.



$$\text{Full load Cu loss, } P_C = \text{Wattmeter reading} = W_S$$
$$\text{Applied voltage} = \text{Voltmeter reading} = V_{SC}$$
$$\text{F.L. primary current} = \text{Ammeter reading} = I_1$$

$$P_C = I_1^2 R_1 + I_1^2 R'_2 = I_1^2 R_{01}$$

$$\therefore \quad R_{01} = \frac{P_C}{I_1^2}$$

where R_{01} is the total resistance of transformer referred to primary.

Total impedance referred to primary, $Z_{01} = \dfrac{V_{SC}}{I_1}$

Total leakage reactance referred to primary, $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$

Short-circuit p.f, $\cos\phi_2 = \dfrac{P_C}{V_{SC} I_1}$

Thus short-circuit lest gives full-load Cu loss, R_{01} and X_{01}.

Losses in a Transformer

The power losses in a transformer are of two types, namely;

1. Core or Iron losses 2. Copper losses

These losses appear in the form of heat and produce (i) an increase in temperature and (ii) a drop in efficiency.

1. Core or Iron losses (Pi)

These consist of hysteresis and eddy current losses and occur in the transformer core due to the alternating flux. These can be determined by open-circuit test. Hysteresis loss, kh $f^{1.6}$ Bm watts /m^3 Eddy current loss ke f^2 B^2 t watts/m^3

Both hysteresis and eddy current losses depend upon (i) maximum flux density Bm in the core and (ii) supply frequency f. Since transformers are connected to constant-frequency, constant voltage supply, both f and Bm are constant. Hence, core or iron losses are practically the same at all loads.

Iron or Core losses, Pi = Hysteresis loss + Eddy current loss = Constant losses

The hysteresis loss can be minimized by using steel of high silicon content whereas eddy current loss can be reduced by using core of thin laminations.

2. Copper losses

These losses occur in both the primary and secondary windings due to their ohmic resistance. These can be determined by short-circuit test.

$$\text{Total Cu losses, } P_C = I_1^2 R_1 + I_2^2 R_2$$
$$= I_1^2 R_{01} \text{ or } I_2^2 R_{02}$$

It is clear that copper losses vary as the square of load current Thus if copper losses are 400 W at a load current of 10 A, then they will be $(1/2)^2 \times 400 = 100$ W at a load current of 5A.

$$\text{Total losses in a transformer} = P_1 + P_C$$
$$= \text{Constant losses} + \text{Variable losses}$$

It may be noted that in a transformer, copper losses account for about 90% of the total losses.

Efficiency of a Transformer

Like any other electrical machine, the efficiency of a transformer is defined as the ratio of output power (in watts or kW) to input power (watts or kW) i.e.,

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}}$$

It may appear that efficiency can be determined by directly loading the transformer and measuring the input power and output power. However, this method has the following drawbacks:

(i) Since the efficiency of a transformer is very high, even 1% error in each wattmeter (output and input) may give ridiculous results. This test, for instance, may give efficiency higher than 100%.

(ii) Since the test is performed with transformer on load, considerable amount of power is wasted. For large transformers, the cost of power alone would be considerable.

(iii) It is generally difficult to have a device that is capable of absorbing all of the output power.

(iv) The test gives no information about the proportion of various losses.

Due to these drawbacks, direct loading method is seldom used to determine the efficiency of a transformer. In practice, open-circuit and short-circuit tests are carried out to find the efficiency.

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

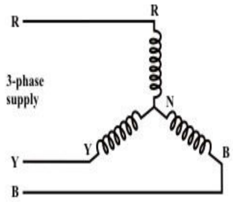
Unit – 4 (BEE) R19&R20 Regulations – I ECE II Semester

Induction Machine: Principle of operation and construction of three-phase induction motors –slip ring and squirrel cage motors – slip-torque characteristics – efficiency calculation – starting methods Brake test on 3-Phase Induction Motor.

CONSTRUCTION OF 3-PHASE INDUCTION MOTOR

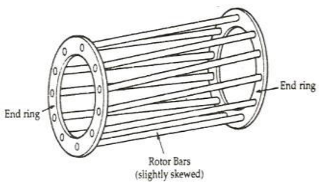
The 3-Phase induction motor consists of mainly two parts namely stator and rotor

Stator: The stator consists of



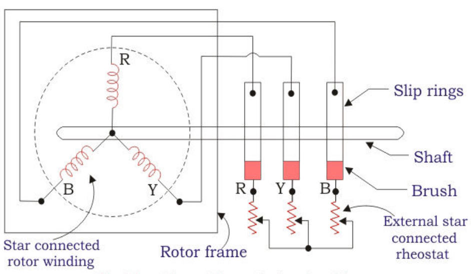
- ***Stator frame:*** The stator frame is made of cast iron and consists of cooling fins
  It gives the support and protects other parts of the motor
- ***Stator core:*** The stator core is made of with laminated high grade alloy steel stampings and slotted on the inner periphery and these stampings are insulated.
- ***Stator winding:*** The stator winding is placed in the stator core, which is connected either in star or delta

Squirrel cage Rotor:



1. The rotor core is a cylindrical one built from a high grade alloy steel laminations.
2. It consists of rotor slots in parallel to the shaft axis on the outer periphery.
3. In general the slots are not parallel to the shaft but skewed with some angle to the shat
4. The purpose of the skewing is to prevent interlocking and to reduce the humming noise
5. The rotor copper bars are placed in the rotor slots and the bars are short circuited with end rings.
6. In Cage rotor type there is no chance of adding the external resistance to the rotor to improve the torque developed at starting.

- **Slip ring rotor (or) Phase Wound rotor:**



Slip Ring Three Phase Induction Motor

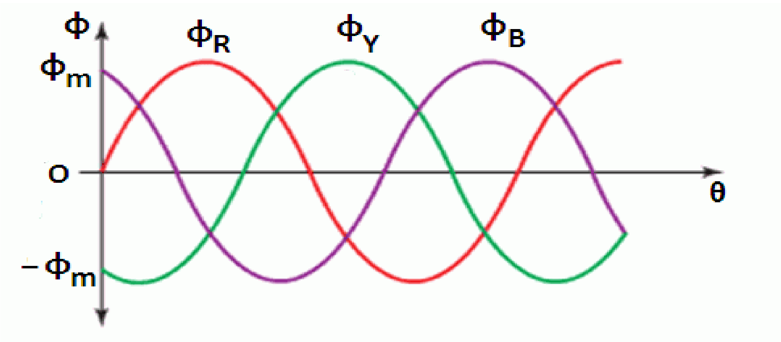
1

1. The rotor core is a cylindrical one built from a high grade alloy steel laminations.
2. It consists of rotor slots on the outer periphery where the star connected winding is done.
3. The star connected rotor winding is done for the same poles as that of the stator winding
4. The ends of the star connected rotor winding are connected to the three slip rings placed on the shaft.
5. The carbon brushes are mounted on the slip rings, through which an external resistance is added to the rotor.
6. The advantage of the Wound rotor is the starting torque is improved by adding the external resistance to the rotor using slip rings.

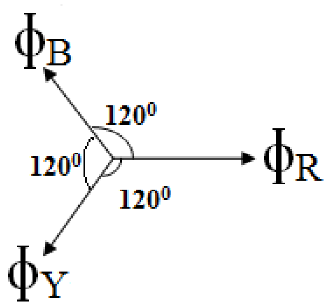
Rotating Magnetic Field

1. The induction motor rotates due to the **rotating magnetic field in 3 phase induction motor**, which is produced by the stator winding in the air gap between in the stator and the rotor.
2. The stator has a three phase stationary winding which can be either star connected or delta connected.
3. Whenever the AC supply is connected to the stator windings, line currents I_R, I_Y, and I_B start flowing.
4. These line currents have phase difference of 120^o with respect to each other.
5. Due to each line current, a sinusoidal flux is produced in the air gap.
6. These fluxes have the same frequency as that of the line currents, and they also have the same phase difference of 120^o with respect to each other.

Let the flux produced by the line currents I_R, I_B, I_Y be φ_R, φ_B, φ_Y respectively.



Fluxes produced by line currents



Mathematically, they are represented as follows:

$\varphi_R = \varphi_m \sin \omega t = \varphi_m \sin \theta$
$\varphi_Y = \varphi_m \sin (\omega t - 120^o) = \varphi_m \sin (\theta - 120^o)$
$\varphi_B = \varphi_m \sin (\omega t - 240^o) = \varphi_m \sin (\theta - 240^o)$

The effective or total flux (ϕ_T) in the air gap is equal to the phasor sum of the three components of fluxes ϕ_R, ϕ_Y and, ϕ_B.

Therefore, $\phi_T = \phi_R + \phi_Y + \phi_B$

Step 1: The values of total flux ϕ_T for different values of θ such as 0, 60, 120 , 180 ….. 360^o. are to be calculated

Step 2: For every value of θ in step 1, draw the phasor diagram, with the phasor ϕ_R as the reference phasor i.e. all the angles are drawn with respect to this phasor.

For θ = 0⁰

$\varphi_R = \varphi_m \sin \omega t = \varphi_m \sin \theta = 0$

$\varphi_Y = \varphi_m \sin(\omega t - 120^o) = \varphi_m \sin(\theta - 120^o) = \varphi_m \sin(0 - 120^o) = (-)\varphi_m \sin 120^o = -0.866\ \varphi_m$

$\varphi_B = \varphi_m \sin(\omega t - 240^o) = \varphi_m \sin(\theta - 240^o) = \varphi_m \sin(0 - 240^o) = (-)\varphi_m \sin 240^o = 0.866\ \varphi_m$

Therefore, $\phi_T = 0 + \phi_Y + \phi_B = \phi_T = 0 + (-\phi_Y) + \phi_B$

$$\Phi_T = \sqrt{(\Phi_Y)^2 + (\Phi_B)^2 + 2\Phi_Y\Phi_B\cos 60}$$

$$\Phi_T = \sqrt{\left(\frac{\sqrt{3}}{2}\varphi_m\right)^2 + \left(\frac{\sqrt{3}}{2}\varphi_m\right)^2 + 2\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)\times\frac{1}{2}}$$

$$\Phi_T = \sqrt{3\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)^2} = \frac{3}{2}\varphi_m = 1.5\varphi_m$$

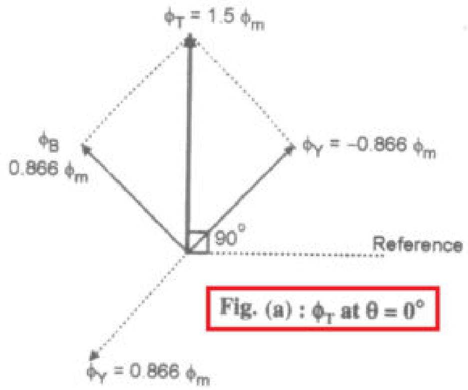
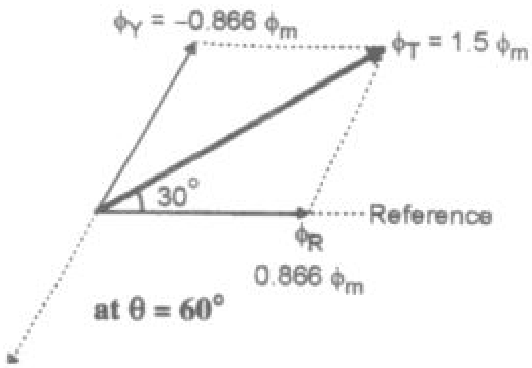


Fig. (a) : ϕ_T at θ = 0°



at θ = 60°

For θ = 60⁰

$\varphi_R = \varphi_m \sin \omega t = \varphi_m \sin \theta = \varphi_m \sin 60 = 0.866\ \varphi_m$

$\varphi_Y = \varphi_m \sin(\omega t - 120^o) = \varphi_m \sin(\theta - 120^o) = \varphi_m \sin(60 - 120^o) = (-)\varphi_m \sin 60^o = -0.866\ \varphi_m$

$\varphi_B = \varphi_m \sin(\omega t - 240^o) = \varphi_m \sin(\theta - 240^o) = \varphi_m \sin(60 - 240^o) = (-)\varphi_m \sin 180^o = 0$

Therefore, $\phi_T = \phi_R + (-\phi_Y) + 0$

$$\Phi_T = \sqrt{(\Phi_R)^2 + (\Phi_Y)^2 + 2\Phi_Y\Phi_R\cos 60}$$

$$\Phi_T = \sqrt{\left(\frac{\sqrt{3}}{2}\varphi_m\right)^2 + \left(\frac{\sqrt{3}}{2}\varphi_m\right)^2 + 2\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)\times\frac{1}{2}}$$

$$\Phi_T = \sqrt{3\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)^2} = \frac{3}{2}\varphi_m = 1.5\varphi_m$$

For θ = 120⁰

$\varphi_R = \varphi_m \sin \omega t = \varphi_m \sin \theta = \varphi_m \sin 120 = 0.866\ \varphi_m$

$\varphi_Y = \varphi_m \sin (\omega t - 120^o) = \varphi_m \sin (120 - 120^o) = (-)\varphi_m \sin 0^o = 0$

$\varphi_B = \varphi_m \sin (\omega t - 240^o) = \varphi_m \sin (120 - 240^o) = (-)\varphi_m \sin 120^o = -0.866\ \varphi_m$

Therefore, $\phi_T = \phi_R + 0 + (-\phi_B)$

$$\Phi_T = \sqrt{(\Phi_R)^2 + (\Phi_B)^2 + 2\Phi_R\Phi_B \cos 60}$$

$$\Phi_T = \sqrt{\left(\frac{\sqrt{3}}{2}\varphi_m\right)^2 + \left(\frac{\sqrt{3}}{2}\varphi_m\right)^2 + 2\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)\times\frac{1}{2}}$$

$$\Phi_T = \sqrt{3\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)^2} = \frac{3}{2}\varphi_m = 1.5\varphi_m$$

For θ = 180⁰

$\varphi_R = \varphi_m \sin \omega t = \varphi_m \sin \theta = \varphi_m \sin 180 = 0$

$\varphi_Y = \varphi_m \sin (\omega t - 120^o) = \varphi_m \sin (180 - 120^o) = \varphi_m \sin 60^o = 0.866\ \varphi_m$

$\varphi_B = \varphi_m \sin (\omega t - 240^o) = \varphi_m \sin (180 - 240^o) = (-)\varphi_m \sin 60^o = -0.866\ \varphi_m$

Therefore, $\phi_T = 0 + \phi_Y + (-\phi_B)$

$$\Phi_T = \sqrt{(\Phi_Y)^2 + (\Phi_B)^2 + 2\Phi_Y\Phi_B \cos 60}$$

$$\Phi_T = \sqrt{\left(\frac{\sqrt{3}}{2}\varphi_m\right)^2 + \left(\frac{\sqrt{3}}{2}\varphi_m\right)^2 + 2\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)\times\frac{1}{2}}$$

$$\Phi_T = \sqrt{3\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)^2} = \frac{3}{2}\varphi_m = 1.5\varphi_m$$

For θ = 240⁰

$\varphi_R = \varphi_m \sin \omega t = \varphi_m \sin \theta = \varphi_m \sin 240 = -0.866\ \varphi_m$

$\varphi_Y = \varphi_m \sin (\omega t - 120^o) = \varphi_m \sin (240 - 120^o) = \varphi_m \sin 120^o = 0.866\ \varphi_m$

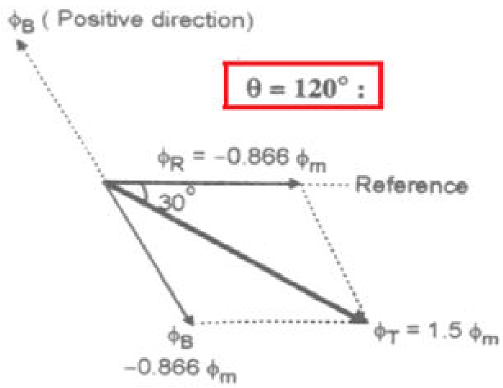
$\varphi_B = \varphi_m \sin (\omega t - 240^o) = \varphi_m \sin (240 - 240^o) = \varphi_m \sin 0^o = 0$

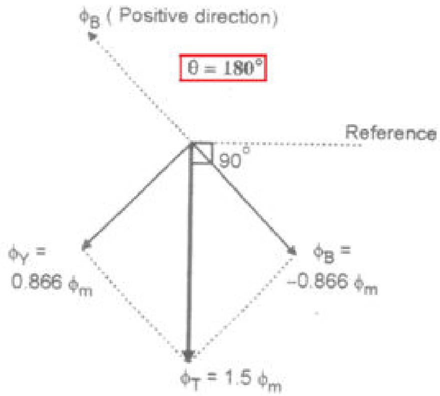
Therefore, $\phi_T = (-\phi_R) + \phi_Y + 0$

$$\Phi_T = \sqrt{(\Phi_R)^2 + (\Phi_Y)^2 + 2\Phi_R\Phi_Y \cos 60}$$

$$\Phi_T = \sqrt{\left(\frac{\sqrt{3}}{2}\varphi_m\right)^2 + \left(\frac{\sqrt{3}}{2}\varphi_m\right)^2 + 2\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)\times\frac{1}{2}}$$

$$\Phi_T = \sqrt{3\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)^2} = \frac{3}{2}\varphi_m = 1.5\varphi_m$$





For θ = 300⁰

$\varphi_R = \varphi_m \sin \omega t = \varphi_m \sin \theta = \varphi_m \sin 300 = -0.866\ \varphi_m$

$\varphi_Y = \varphi_m \sin(\omega t - 120^o) = \varphi_m \sin(300 - 120^o) = \varphi_m \sin 180^o = 0$

$\varphi_B = \varphi_m \sin(\omega t - 240^o) = \varphi_m \sin(300 - 240^o) = \varphi_m \sin 60^o = 0.866\ \varphi_m$

Therefore, $\phi_T = (-\phi_R) + \phi_Y + 0$

$$\Phi_T = \sqrt{(\Phi_R)^2 + (\Phi_B)^2 + 2\Phi_R\Phi_B \cos 60}$$

$$\Phi_T = \sqrt{\left(\frac{\sqrt{3}}{2}\varphi_m\right)^2 + \left(\frac{\sqrt{3}}{2}\varphi_m\right)^2 + 2\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)\times\frac{1}{2}}$$

$$\Phi_T = \sqrt{3\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)^2} = \frac{3}{2}\varphi_m = 1.5\varphi_m$$

For θ = 360⁰

$\varphi_R = \varphi_m \sin \omega t = \varphi_m \sin \theta = \varphi_m \sin 360 = 0$

$\varphi_Y = \varphi_m \sin(\omega t - 120^o) = \varphi_m \sin(360 - 120^o) = \varphi_m \sin 240^o = -\ 0.866\ \varphi_m$

$\varphi_B = \varphi_m \sin(\omega t - 240^o) = \varphi_m \sin(300 - 240^o) = \varphi_m \sin 60^o = 0.866\ \varphi_m$

Therefore, $\phi_T = 0 + (-\phi_Y) + \phi_B$

$$\Phi_T = \sqrt{(\Phi_Y)^2 + (\Phi_B)^2 + 2\Phi_Y\Phi_B \cos 60}$$

$$\Phi_T = \sqrt{\left(\frac{\sqrt{3}}{2}\varphi_m\right)^2 + \left(\frac{\sqrt{3}}{2}\varphi_m\right)^2 + 2\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)\times\frac{1}{2}}$$

$$\Phi_T = \sqrt{3\times\left(\frac{\sqrt{3}}{2}\varphi_m\right)^2} = \frac{3}{2}\varphi_m = 1.5\varphi_m$$

In the similar way as shown in the phasor diagrams the resultant or total flux rotates 60 degrees for every instant and completes one cycle of rotation in the direction of phase sequence of the supply.

Thus when a three phase supply is applied to the three phase winding connected either in star or delta it produces a rotating magnetic field having

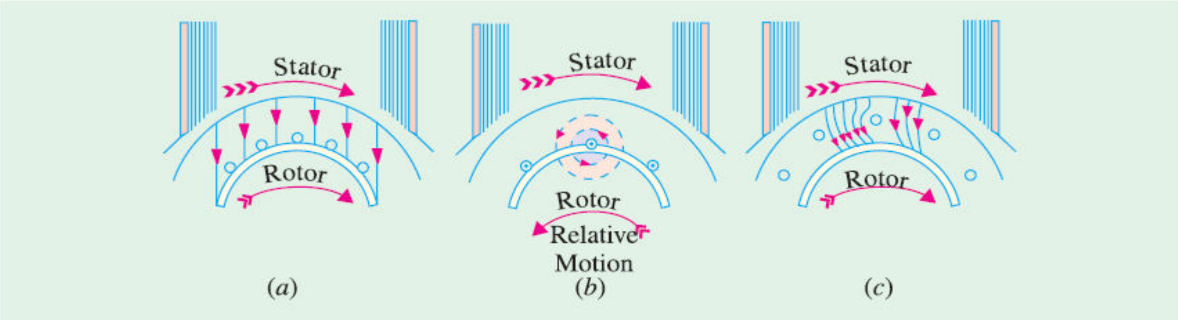
i. ***a constant magnitude of 1.5 times the Φ_m***
ii. ***a constant speed of synchronous speed N_s=120f/P***
iii. ***a direction equal to its phase sequence***

WORKING PRINCIPLE OF 3-PHASE INDUCTION MOTOR

1. The balanced three-phase winding of the stator is supplied with a balanced three-phase voltage.
2. The current in the stator winding produces a rotating magnetic field, with constant magnitude of $1.5Ø_m$ and rotates at synchronous speed of N_s=120f/P
3. The magnetic flux lines in the air gap cut both stator and rotor (being stationary, as the motor speed is zero) conductors at the same speed.
4. The emfs in both stator and rotor conductors are induced at the same frequency, i.e. line or supply frequency, with No. of poles for both stator and rotor windings (assuming wound one) being same.
5. As the rotor winding is short-circuited at the slip-rings, current flows in the rotor windings.
6. The electromagnetic torque in the motor is in the same direction as that of the rotating magnetic field, due to the interaction between the rotating flux produced in the air gap by the current in the stator winding, and the current in the rotor winding.
7. This is as per Lenz's law, as the developed torque is in such direction that it will oppose the cause, which results in the current flowing in the rotor winding.
8. As the rotor starts rotating in the same direction, as that of the rotating magnetic field due to production of the torque as stated earlier, the relative velocity decreases, along with lower values of induced emf and current in the rotor.
9. If the rotor speed is equal that of the rotating magnetic field, which is termed as synchronous speed, and also in the same direction, the relative velocity is zero, which causes both the induced emf and current in the rotor to be reduced to zero. Under this condition, torque will not be produced.
10. So, for production of positive (motoring) torque, the rotor speed must always be lower than the synchronous speed. The rotor speed is never equal to the synchronous speed in an IM.

The setting up of the torque for rotating the rotor is explained below :

In Fig (a) is shown the stator field which is assumed to be rotating clockwise. The relative motion of the rotor with respect to the stator is ***anticlockwise***. By applying Right-hand rule, the direction of the induced e.m.f. in the rotor is found to be outwards. Hence, the direction of the flux due to rotor current *alone*, is as shown in Fig. (b). Now, by applying the Left-hand rule, or by the effect of combined field [Fig. (c)] it is clear that the rotor conductors experience a force tending to rotate them in clockwise direction. Hence, the rotor is set into rotation in the same direction as that of the stator flux (or field).



Slip

It is defined as the relative speed or slip speed (N_s - N_r) expressed in terms of synchronous speed

$$s = \frac{N_S - N_r}{N_s}$$

- Since the speed at standstill is zero, the slip is 1.
- As the speed of rotor increases the slip decreases.

EFFECT OF ROTOR QUANTITIES WITH RESPECT TO THE SLIP IN A 3-PHASE INDUCTION MOTOR

The rotor quantities that effect with slip are

1. Rotor induced emf (E_r)
2. rotor emf's frequency (f_r)
3. induced currents (I_r)
4. rotor power factor ($\cos\Phi_r$)

Rotor induced emf

Under running conditions the induced emf is directly proportional to the relative speed (N_s-N_r)

E_2 at standstill = K N_s,

E_r at running = K (N_s-N_r)

$$\frac{E_r}{E_2} = \frac{N_S - N_r}{N_s} = s$$

Therefore $E_r = sE_2$

Rotor emf's frequency

Frequency of rotor emf at stand still $f_2=PN_s/120$

Frequency of rotor emf at running $f_r=P(N_s-N_r)/120$

$$\frac{f_r}{f_2} = \frac{N_S - N_r}{N_s} = s \qquad \text{Therefore } f_r = sf_2 = sf$$

Rotor reactance

Rotor reactance at stand still $X_2 = 2\Pi f L_2$

Rotor reactance at running $X_r = 2\Pi f_r L_2$

$$\frac{X_r}{X_2} = \frac{2\pi f_r L_2}{2\pi f_2 L_2} = \frac{f_r}{f_2} = \frac{sf_2}{f_2} = s \qquad \text{Therefore } X_r = sX_2$$

Rotor impedance

Rotor impedance at stand still $Z_2 = R_2 + jX_2$

Rotor impedance at running $Z_r = R_2 + jX_r$

$$Z_2 = \sqrt{R_2^2 + X_2^2} \quad \text{and} \quad Z_r = \sqrt{R_2^2 + X_r^2} = \sqrt{R_2^2 + (sX_2)^2}$$

Rotor induced currents

The rotor current is defined as the ratio of the rotor emf to the rotor impedance.

Rotor current at stand still is

$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{r_2^2 + x_2^2}}$$

Rotor current at running is

$$I_r = \frac{E_r}{Z_r} = \frac{sE_2}{\sqrt{r_2^2 + (s^2 x_2^2)}}$$

Rotor power factor

Power factor is defined as the ratio of the rotor resistance to the rotor impedance

Rotor power factor at stand still is

$$\cos\phi_2 = \frac{r_2}{Z_2} = \frac{r_2}{\sqrt{r_2^2 + x_2^2}}$$

Rotor power factor at running is

$$\cos\phi_r = \frac{r_2}{Z_r} = \frac{r_2}{\sqrt{r_2^2 + (s^2 x_2^2)}}$$

TORQUE EQUATION OF 3-Φ INDUCTION MOTOR

Torque at stand still

The torque in the motor is directly proportional to the product of flux and active component of the rotor current

$$T \,\alpha\, \phi I_2 \cos\phi_2$$

Here the flux is directly proportional to the rotor induced emf E_2 i.e $\phi \,\alpha\, E_2$

The rotor current I_2 and rotor power factor $\cos\Phi_2$ are

$$I_2 = \frac{E_2}{Z_2} \text{ and } \cos\phi_2 = \frac{R_2}{Z_2}$$

$$T \,\alpha\, E_2 \times \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

$$T = k\frac{E_2^2 R_2}{R_2^2 + X_2^2} \text{ where } k = \frac{3}{2\pi n_s} \text{ here } n_s \text{ is synchronous speed in rps}$$

Therefore, torque at standstill is $T = \left(\frac{3}{2\pi n_s}\right)\frac{E_2^2 R_2}{R_2^2 + X_2^2}$

From the above equation torque at stand still depends on rotor resistance (R_2), so keeping this R_2 as variable the condition for the maximum torque at standstill is

$$\frac{dT_{st}}{dR_2} = 0$$

Rewriting the stand still torque

$$T = K\frac{R_2}{R_2^2 + X_2^2} \text{ where } K = \frac{3E_2^2}{2\pi n_s} \text{ and } T \,\alpha\, \frac{R_2}{R_2^2 + X_2^2}$$

$$\frac{d\left(\frac{R_2}{R_2^2 + X_2^2}\right)}{dR_2} = 0 \quad \Rightarrow \frac{\left(R_2^2 + X_2^2\right) - R_2\left(2R_2\right)}{\left(R_2^2 + X_2^2\right)^2} = 0 \quad \Rightarrow \left(R_2^2 + X_2^2\right) - R_2\left(2R_2\right) = 0$$

$$R_2^2 + X_2^2 = 2R_2^2 \;\Rightarrow R_2^2 = X_2^2 \quad \Rightarrow R_2 = X_2$$

Therefore on adding the resistance to the rotor such that $R_2 = X_2$, the motor will develop maximum torque at stand still.

The maximum torque at standstill is $T_{max} = \left(\frac{3}{2\pi n_s}\right)\frac{E_2^2 X_2}{X_2^2 + X_2^2} = \frac{K}{2X_2}$ $\quad T_{max} = \frac{3}{2\pi n_s}\frac{E_2^2}{2X_2}$

Torque at running condition

The torque in the motor is directly proportional to the product of flux and active component of the rotor current

$$T \,\alpha\, \phi\, I_r \cos\phi_r$$

Here the flux is directly proportional to the rotor induced emf E_2 i.e $\phi \,\alpha\, E_2$

The rotor current I_r and rotor power factor $\cos\Phi_r$ are

$$I_r = \frac{E_r}{Z_r} \text{ and } \cos\phi_r = \frac{R_2}{Z_r}$$

$$T \,\alpha\, E_2 \times \frac{E_r}{\sqrt{R_2^2 + X_r^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_r^2}}$$

$$T \,\alpha\, E_2 \times \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$T = k\frac{sE_2^2R_2}{R_2^2+(sX_2)^2} \quad \text{where} \quad k=\frac{3}{2\pi n_s} \quad \text{here } n_s \text{ is synchronou s speed in rps}$$

Therefore, torque at standstill is $T=\left(\frac{3}{2\pi n_s}\right)\frac{sE_2^2R_2}{R_2^2+(sX_2)^2}$

From the above equation torque under running condition depends on slip (s), so keeping this 's' as variable the condition for the maximum torque at running is

$$\frac{dT}{ds}=0$$

Rewriting the torque under running

$$T = K\frac{sR_2}{R_2^2+(sX_2)^2} \quad \text{where } K=\frac{3E_2^2}{2\pi n_s} \quad \text{and } T\,\alpha\,\frac{sR_2}{R_2^2+(sX_2)^2}$$

$$\frac{d\left(\frac{sR_2}{R_2^2+(sX_2)^2}\right)}{ds}=0 \quad \Rightarrow \frac{\left(R_2^2+(sX_2)^2\right)R_2 - sR_2\left(2sX_2^2\right)}{\left(R_2^2+(sX_2)^2\right)^2}=0 \quad \Rightarrow \left(R_2^2+(sX_2)^2\right)R_2 - sR_2\left(2sX_2^2\right)=0$$

$$\left(R_2^2+(sX_2)^2\right)=s\left(2sX_2^2\right) \quad \Rightarrow R_2^2+(sX_2)^2=2s^2X_2^2 \quad \Rightarrow R_2^2=(sX_2)^2 \quad \Rightarrow R_2=(sX_2) \Rightarrow s_m=\frac{R_2}{X_2}$$

Therefore the motor when rotates at a slip $s_m=\frac{R_2}{X_2}$ then the motor will develop maximum torque at running.

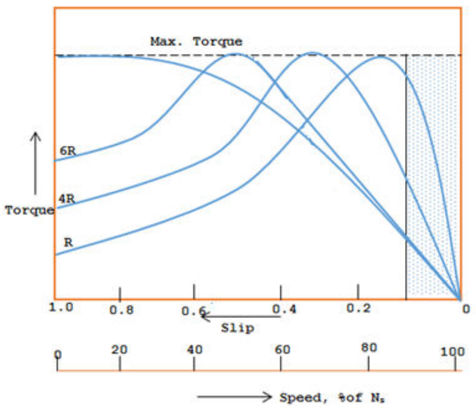
The maximum torque at running is

$$T = K\frac{s^2X_2}{(sX_2)^2+(sX_2)^2} \quad \text{where } K=\frac{3E_2^2}{2\pi n_s}$$

$$T = K\frac{s^2X_2}{2(sX_2)^2} = \frac{K}{2X_2} \qquad T_{max}=\frac{3}{2\pi n_s}\frac{E_2^2}{2X_2}$$

Thus, the magnitude of the maximum torque is same at both standstill and running conditions

TORQUE - SLIP CHARACTERISTICS



- The torque-slip characteristics in an induction motor shows the variation of the torque developed with respect to changes of slip.
- When the load on the motor is removed gradually the speed increases and the slip

decreases

- Considering, the speed at standstill $N_r = 0$ the slip s =1 and as the speed increases from 0 to N_s the slip s decreases from 1 to zero, any how the induction motor never rotates at N_s so the slip never becomes 0
- Let the torque in an induction motor is

$$T = \left(\frac{3}{2\pi n_s}\right)\frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \quad T\alpha \frac{sR_2}{R_2^2 + (sX_2)^2}$$

- For the smaller values of slips i.e $0 < s < s_m$, $sX_2 << R_2$ so neglecting sX_2 , the torque in this smaller range of slips is

$$T\alpha \frac{sR_2}{R_2^2} \quad T\alpha \frac{s}{R_2} \quad T\alpha\, s$$

- As the torque is directly proportional to slip s, Therefore as slip increases the torque increases linearly and attains maximum torque when slip $s = s_m$
- For the larger values of slips i.e $s_m < s < 1$, R2 << sX_2 so neglecting R_2 , the torque in this larger range of slips is

$$T\alpha \frac{R_2}{sX_2{}^2} \quad T\alpha \frac{1}{s}$$

- As the torque is inversely proportional to slip s, Therefore as slip increases the torque decreases linearly and falls to the value of standstill torque T_{st} at s = 1

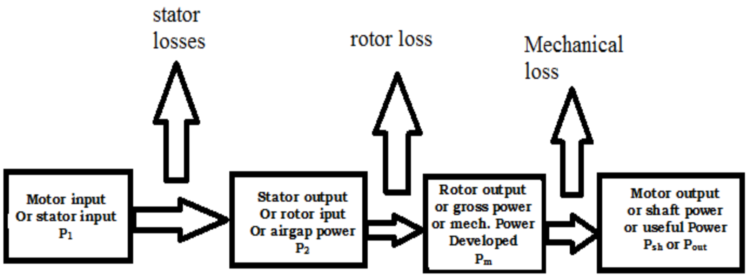
Salient points:

1. The maximum value of the torque is independent to the rotor resistance
2. The slip at which the maximum torque occurs (s_m) depends on the rotor resistance
3. The motor develops maximum torque at starting itself by making s_m =1 which is possible when $R_2 = X_2$

POWER STAGES IN A 3-PHASE INDUCTION MOTOR

In a 3-Phase induction motor the power losses occurs in stator and rotor

- Stator losses = Stator core loss + stator Cu loss
- Rotor losses = Rotor Cu loss
- Mechanical loss = Friction and windage loss



Therefore,

- Stator output power (P_2) = Stator input power (P_1) – stator losses
- Rotor output power (P_m) = Rotor input power (P_2) – rotor Cu loss
- Motor output power (P_{sh}) = Rotor output power (P_m) – Mechanical loss

Relationship between rotor input, rotor output and slip in a 3-Phase induction motor

The mechanical power in a motor is given by

$$P_m = \frac{2\pi N_r}{60}T \text{ or } P_2 = \frac{2\pi N_s}{60}T$$

Where P_m is the mechanical or gross output power and P2 is the air gap power or rotor input. T = torque in the motor

$$P_m = \omega_r T \text{ ---- (1)} \qquad \text{and} \qquad P_2 = \omega_s T \text{ ---- (2)}$$

Eq (2) – Eq (1) = P_2 - P_m = $\omega_s T - \omega_r T = (\omega_s - \omega_r)T$ ---- (3)

Divide Eq (3) with Eq (2)

$$\frac{P_2 - P_m}{P_2} = \frac{\text{Rotor Cu Loss}}{P_2} = \frac{(\omega_s - \omega_r)}{\omega_s} = s$$

Rotor cu loss (RCL) = sP_2 ----- (4)

Divide Eq (1) with Eq (2)

$$\frac{P_m}{P_2} = \frac{\omega_r}{\omega_s} = \frac{P_2 - RCL}{P_2} = \frac{P_2 - sP_2}{P_2} = \frac{P_2(1-s)}{P_2} = 1 - s$$

P_m = (1-s) P_2 ---- (5)

Divide Eq (5) with Eq (4)

$$\frac{P_m}{RCL} = \frac{P_2(1-s)}{sP_2} = \frac{(1-s)}{s}$$

$$P_m = \frac{(1-s)}{s} \times RCL \qquad \text{---- (6)}$$

Finally from Eq's (4), (5) & (6) ***P_2 : RCL:: P_m = 1 : s :: (1-s)***

Thus,

$$\eta = \frac{motor\ output}{motor\ input} = \frac{motor\ output}{motor\ output + mech\ loss + rotor\ cu\ loss + stator\ core\ loss + stator\ cu\ loss}$$

12

STARTING METHOD FOR INDUCTION MOTORS

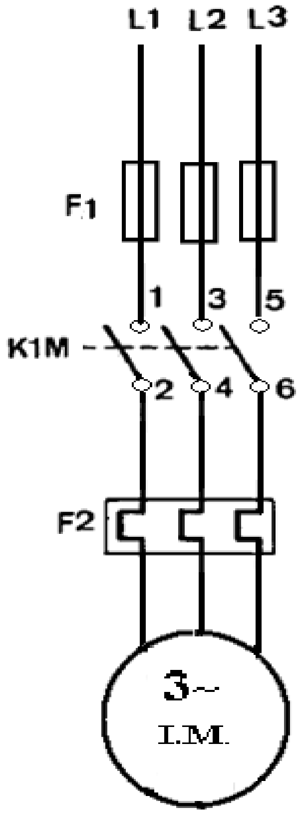
- A 3-phase induction motor is theoretically self starting. The stator of an induction motor consists of 3-phase windings, which when connected to a 3-phase supply creates a rotating magnetic field. This will link and cut the rotor conductors which in turn will induce a current in the rotor conductors and create a rotor magnetic field. The magnetic field created by the rotor will interact with the rotating magnetic field in the stator and produce rotation.
- Therefore, 3-phase induction motors employ a starting method not to provide a starting torque at the rotor, but because of the following reasons;

  1. Reduce heavy starting currents and prevent motor from overheating.
  2. Provide overload and no-voltage protection.

- There are many methods in use to start 3-phase induction motors. Some of the common methods are;

  - Direct On-Line Starter (DOL)
  - Star-Delta Starter
  - Auto Transformer Starter
  - Rotor resistance Starter

<u>Direct On-Line Starter (DOL)</u>

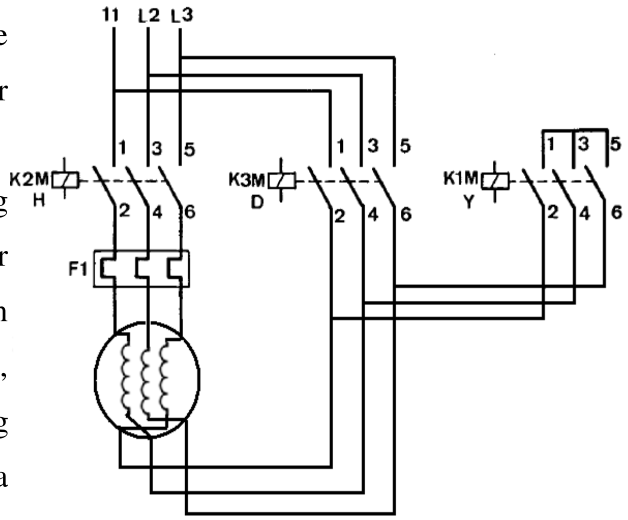


1. The Direct On-Line (DOL) starter is the simplest and the most inexpensive of all starting methods and is usually used for squirrel cage induction motors.
2. It directly connects the contacts of the motor to the full supply voltage. The starting current is very large, normally 6 to 8 times the rated current.
3. The starting torque is likely to be 0.75 to 2 times the full load torque.
4. In order to avoid excessive voltage drops in the supply line due to high starting currents, the DOL starter is used only for motors with a rating of less than 5KW
5. There are safety mechanisms inside the DOL starter which provides protection to the motor as well as the operator of the motor.
6. The DOL starter consists of a coil operated contactor K1M controlled by start and stop push buttons.

7. On pressing the start push button S1, the contactor coil K1M is energized from line L1. The three mains contacts (1-2), (3-4), and (5-6) in fig. are closed. The motor is thus connected to the supply.
8. When the stop push button S2 is pressed, the supply through the contactor K1M is disconnected. Since the K1M is de-energized, the main contacts (1- 2), (3-4), and (5-6) are opened. The supply to motor is disconnected and the motor stops.

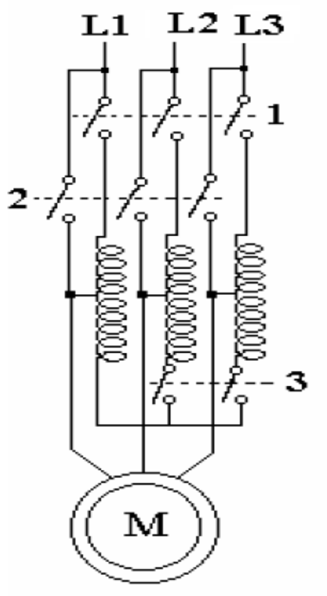
Star-Delta Starter

1. The star delta starting is a very common type of starter and extensively used, compared to the other types of the starters. This method used reduced supply voltage in starting. Figure shows the connection of a 3phase induction motor with a star – delta starter.



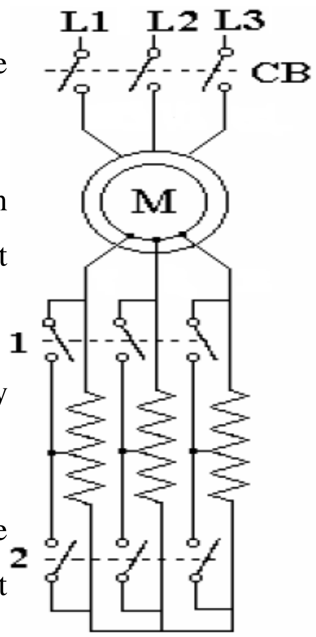
2. The method achieved low starting current by first connecting the stator winding in star configuration, and then after the motor reaches a certain speed, throw switch changes the winding arrangements from star to delta configuration.
3. By connecting the stator windings, first in star and then in delta, the line current drawn by the motor at starting is reduced to one-third as compared to starting current with the windings connected in delta.
4. At the time of starting when the stator windings are start connected, each stator phase gets voltage $\frac{V_L}{\sqrt{3}}$ where V_L is the line voltage.
5. Since the torque developed by an induction motor is proportional to the square of the applied voltage, star- delta starting reduced the starting torque to one – third that obtainable by direct delta starting.
    a. K2M Main Contactor
    b. K3M Delta Contactor
    c. K1M Star Contactor
    d. F1 Thermal Overload Relay

Auto Transformer Starter



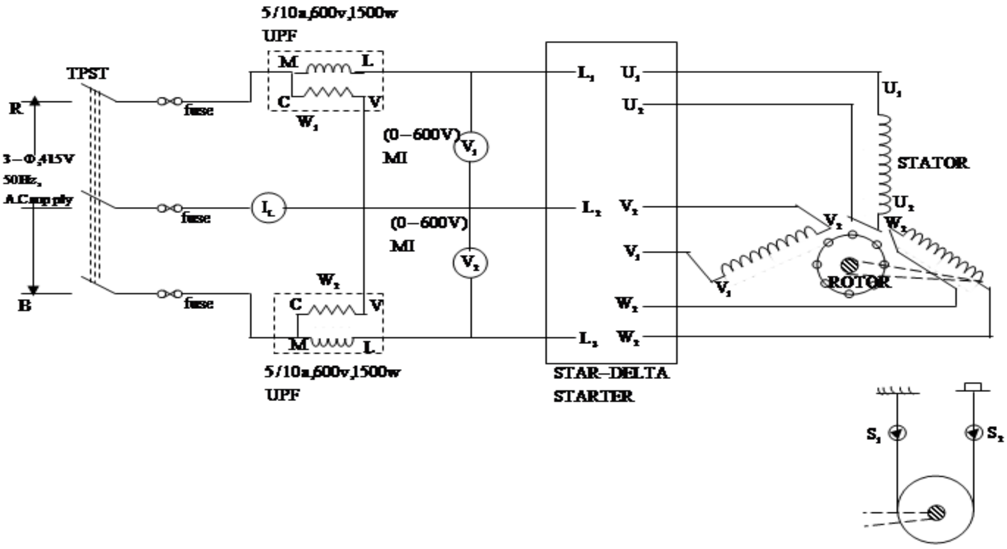
1. The operation principle of auto transformer method is similar to the star delta starter method.
2. The starting current is limited by (using a three phase auto transformer) reduce the initial stator applied voltage.
3. The auto transformer starter is more expensive, more complicated in operation and bulkier in construction when compared with the star – delta starter method.
4. But an auto transformer starter is suitable for both star and delta connected motors, and the starting current and torque can be adjusted to a desired value by taking the correct tapping from the auto transformer.
5. When the star delta method is considered, voltage can be adjusted only by factor of $\frac{1}{\sqrt{3}}$.

Rotor Resistance Starter



1. This method allows external resistance to be connected to the rotor through slip rings and brushes.
2. Initially, the rotor resistance is set to maximum and is then gradually decreased as the motor speed increases, until it becomes zero.
3. The rotor impedance starting mechanism is usually very bulky and expensive when compared with other methods.
4. It also has very high maintenance costs. Also, a considerable amount of heat is generated through the resistors when current runs through them.

BRAKE TEST ON 3-PHASE INDUCTION MOTOR.



1. The brake test is a direct method of testing. It consists of applying a brake to a water – cooled pulley mounted on the shaft of the motor.
2. A rope is wound round the pulley and its two ends are attached to two spring balances S_1 and S_2.
3. The tension of the rope can be adjusted with the help of swivels. Then, the force acting tangentially on the pulley = $(S_1 - S_2)$ Kgs.
4. If r is the pulley radius, the torque at the pulley, $T_{sh} = (S_1 - S_2)\, r$ kg m.
5. If "ω" is the angular velocity of the motor. $\omega = 2\pi N/60$, Where N is the speed in rpm.
6. Motor output $P_{out} = 9.81 \times 2\pi N (s_1 - s_2)\, r$ watts.
7. The motor input can be measured directly from the wattmeter's by summing w_1 and w_2 readings
8. Thus, the efficiency is calculated by taking the ratio of the motor output to the motor input.

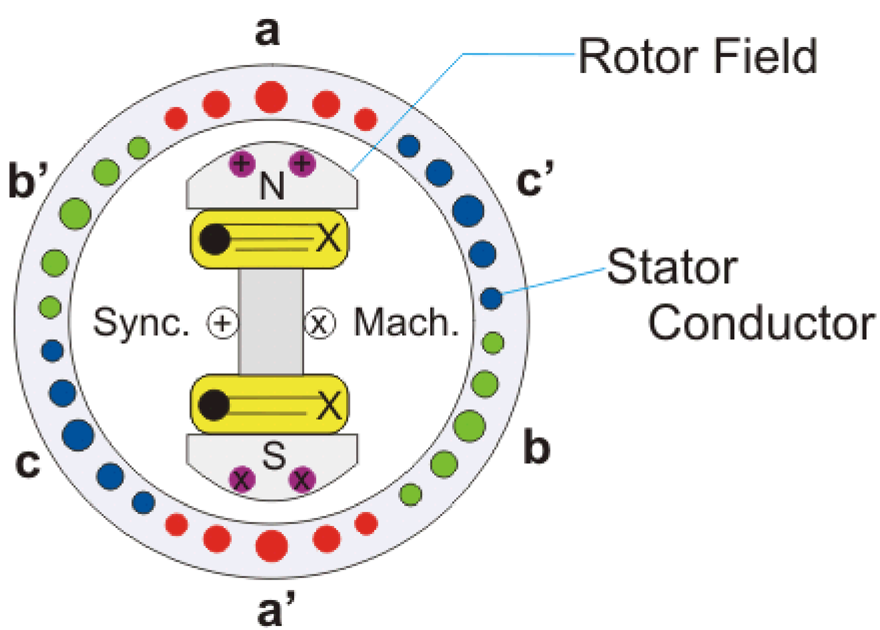
UNIT-IV

ALTERNATORS

Construction of Alternator:

Construction wise, an alternator consists of field poles placed on the rotating fixture of the machine i.e. rotor as shown in the figure above. The rotor rotates in the stator. The field poles get projected on the rotor body. The armature conductors are housed on the stator. An alternating three-phase voltage represented by aa’, bb’, cc’ is induced in the armature conductors thus resulting in the generation of three-phase electrical power. All modern electrical power generating stations use this technology for generation of three-phase power, and as a result, the alternator or synchronous generator has become a subject of great importance and interest for power engineers.

An alternator is basically a type of AC generator which also known as synchronous generator. The field poles are made to rotate at synchronous speed $N_s = 120\ f/P$ for effective power generation. Where, f signifies the alternating current frequency and the P represents the number of poles.



In most practical construction of alternator, it is installed with a stationary armature winding and a rotating field unlike in the case of DC generator where the arrangement is exactly opposite. This modification is made to cope with the very high power of the order of few 100 Megawatts produced in an AC generator contrary to that of a DC generator. To accommodate such high power the conductor weighs and dimensions naturally have to be increased for optimum performance. For this reason is it beneficial to replace these high power armature windings by low power field windings, which is also consequently of much lighter weight, thus reducing the centrifugal force required to turn the rotor and permitting higher speed limits.

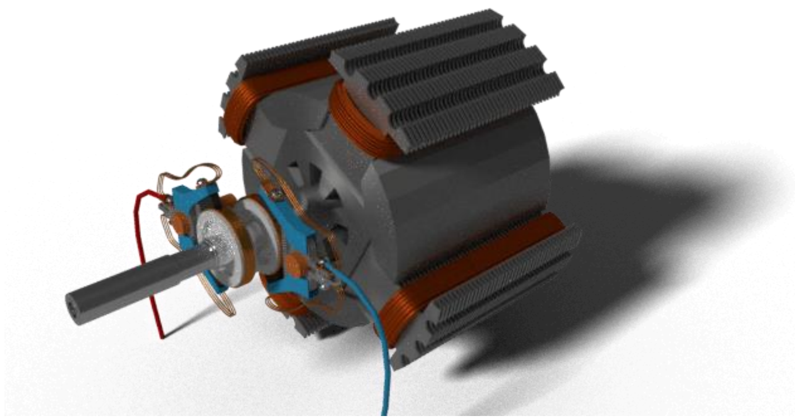
There are mainly two types of rotor used in construction of alternator,

1. Salient pole type.
2. Cylindrical rotor type.

Salient Pole Type

The term salient means protruding or projecting. The salient pole type of rotor is generally used for slow speed machines having large diameters and relatively small axial lengths. The poles, in this case, are made of thick laminated steel sections riveted together and attached to a rotor with the help of joint.

An alternator as mentioned earlier is mostly responsible for generation of very high electrical power. To enable that, the mechanical input given to the machine in terms of rotating torque must also be very high. This high torque value results in oscillation or hunting effect of the alternator or synchronous generator. To prevent these oscillations from going beyond bounds the damper winding is provided in the pole faces as shown in the figure. The damper windings are basically copper bars short-circuited at both ends are placed in the holes made in the pole axis. When the alternator is driven at a steady speed, the relative velocity of the damping winding with respect to the main field will be zero. But as soon as it departs from the synchronous speed there will be relative motion between the damper winding and the main field which is always rotating at synchronous speed. This relative difference will induce the current in them which will exert a torque on the field poles in such a way as to bring the alternator back to synchronous speed operation.



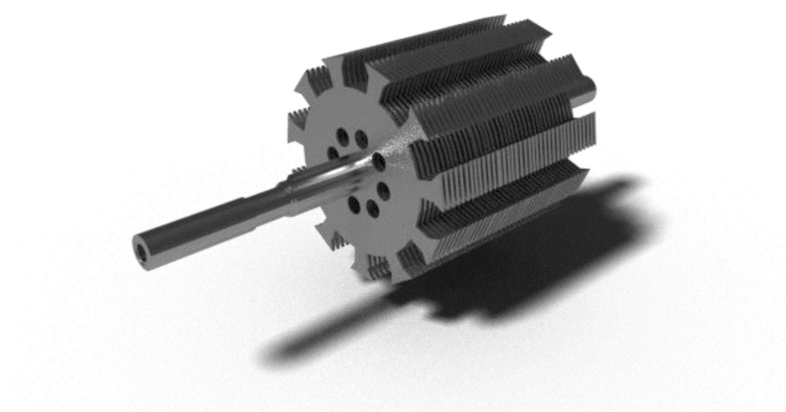
The salient feature of pole field structure has the following special feature-

1. They have a large horizontal diameter compared to a shorter axial length.
2. The pole shoes covers only about $^{2}/_{3}$rd of pole pitch.
3. Poles are laminated to reduce eddy current loss.
4. The salient pole type motor is generally used for low-speed operations of around 100 to 400 rpm, and they are used in power stations with hydraulic turbines or diesel engines.

Salient pole alternators driven by water turbines are called hydro-alternators or hydro generators.

Cylindrical Rotor Type

The cylindrical rotor is generally used for very high speed operation and employed in steam turbine driven alternators like turbo generators. The machines are built in a number of ratings from 10 MVA to over 1500 MVA. The cylindrical rotor type machine has a uniform length in all directions, giving a cylindrical shape to the rotor thus providing uniform flux cutting in all directions. The rotor, in this case, consists of a smooth solid steel cylinder, having a number of slots along its outer periphery for hosting the field coils.



The cylindrical rotor alternators are generally designed for 2-pole type giving very high speed of

$$N_s = \frac{(120 \times f)}{P} = \frac{(120 \times 50)}{2} = 3000\ rpm$$

Or 4-pole type running at a speed of

$$N_s = \frac{(120 \times f)}{P} = \frac{(120 \times 50)}{4} = 1500\ rpm$$

Where, f is the frequency of 50 Hz. The cylindrical rotor synchronous generator does not have any projections coming out from the surface of the rotor, rather central polar area is provided with slots for housing the field windings as we can see from the diagram above. The field coils are so arranged around these poles that flux density is maximum on the polar central line and gradually falls away as we move out towards the periphery. The cylindrical rotor type machine gives better balance and quieter-operation along with lesser windage losses.

Armature Winding of Alternator

Armature winding in an alternator may be either closed type open type. Closed winding forms star connection in armature winding of alternator.There are some common properties of armature winding.

1. First and most important property of an armature winding is, two sides of any coil should be under two adjacent poles. That means, coil span = pole pitch.
2. The winding can either be single layer or double layer.
3. Winding is so arranged in different armature slots, that it must produce sinusoidal emf.

Types of Armature Winding of Alternator

There are different types of armature winding used in alternator. The windings can be classified as

1. Single phase and poly phase armature winding.
2. Concentrated winding and distributed winding.
3. Half coiled and whole coiled winding.
4. Single layer and double layer winding.
5. Lap, wave and concentric or spiral winding and
6. Full pitched coil winding and fractional pitched coil winding.

In addition to these, armature winding of alternator can also integral slot winding and fractional slot winding.

Single Phase Armature Winding

Single phase armature winding can be either concentrated or distributed type.

Concentrated Armature Winding

The concentrated winding is employed where the number of slots on the armature is equal to the number of poles in the machine. This armature winding of alternator gives maximum output voltage but not exactly sinusoidal.

The most simple single-phase winding is shown below in the figure-1. Here, number poles = the number of slots = number of coil sides. Here, one coil side is inside one slot under one pole and the other coil side inside other slots under next pole. The emf induced in one coil side gets added to that of adjacent coil side.

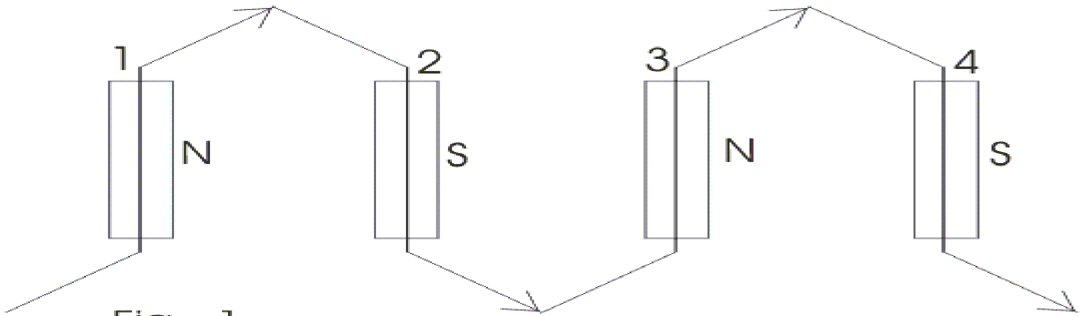


Fig - 1

This arrangement of an armature winding in an alternator is known as skeleton wave winding. As per the fig-1, coil side-1 under N-pole is connected to coil side-2 under S-pole at the back and coil side-3 at the front and so on. The direction of induced emf of coil side-1 is upward and emf induced in coil side-2 is downward. Again as coil side-3 is under N-pole, it will have emf in the upward direction and so on. Hence total emf is the summation of emf of all coil sides. This form of armature winding is quite simple but rarely used as this requires considerable space for end connection of every coil side or conductor. We can overcome this problem, some extent by using multi turns coil. We use the multi-turn half coiled winding to get higher emf. Since the coils cover only one half of the armature periphery thus, we refer this winding as Half coiled or Hemi - tropic winding. Figure - 2 shows this. If we distribute the all coils over the whole armature periphery, then the armature winding is referred as whole coiled winding.

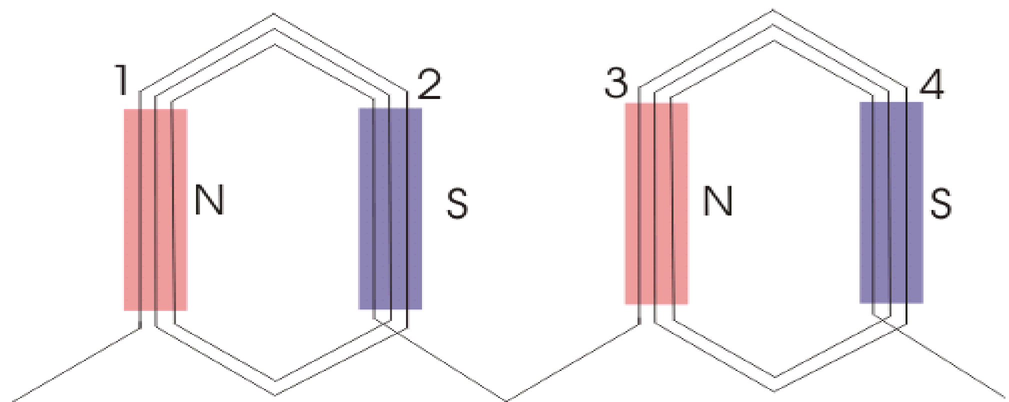


Fig - 2

Figure 3 shows a double layer winding, where we place one side of each coil on the top of armature slot, and another side in the bottom of the slot. (Represented by dotted lines).

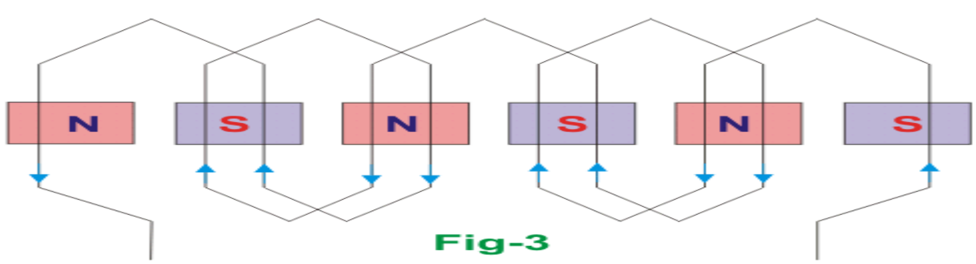


Fig-3

Distributed Armature Winding of Alternator

For obtaining smooth sinusoidal emf wave from, conductors are placed is several slots under single pole. This armature winding is known as distributed winding. Although distributed armature winding in alternator reduces emf, still it is very much usable due to following reason.

1. It also reduces harmonic emf and so waveform is improved.
2. It also diminishes armature reaction.
3. Even distribution of conductors, helps for better cooling.
4. The core is fully utilized as the conductors are distributed over the slots on the armature periphery.

Lap Winding of Alternator

Full pitched lap winding of 4 poles, 12 slots, 12 conductors (one conductor per slot) alternator is shown below. The back pitch of the winding is equal to the number of conductors per pole, i.e., = 3 and the front pitch is equal to back pitch minus one. The winding is completed per pair of the pole and then connected in series as shown in figure - 4 below.

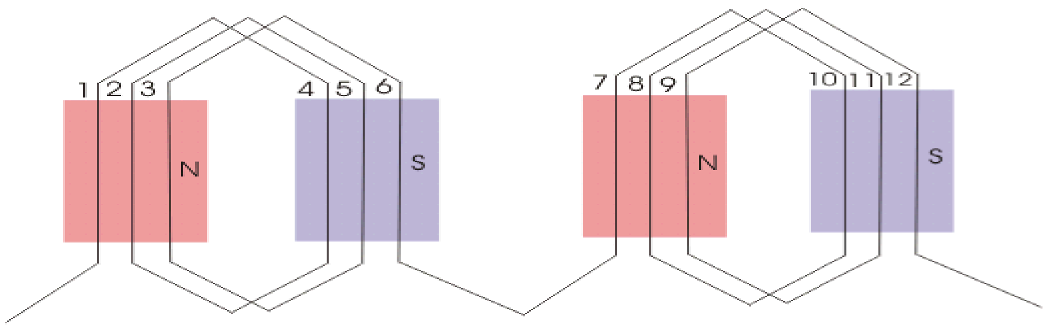


Fig - 4

Wave Winding of Alternator

Wave winding of the same machine, i.e., four poles, 12 slots, 12 conductors is shown in the figure-e below. Here, back pitch and front pitch both equal to some conductor per pole.

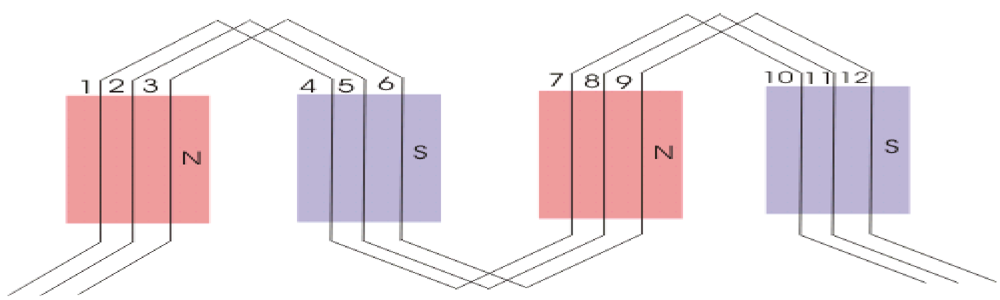


Fig - 5

Concentric or Spiral Winding

This winding for the same machine, i.e., four poles 12 slots 12 conductors alternator is shown in the figure-f below. In this winding, the coils are of different pitches. The outer coil pitch is 5, the middle coil pitch is 3, and inner coil pitch is one.

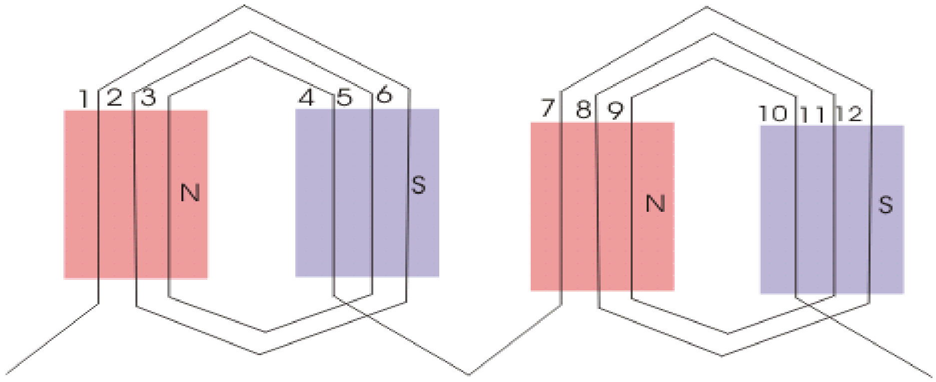


Fig - 6

Poly Phase Armature Winding of Alternator

Before discussing poly phase armature winding of alternator, we should go through some of the related terms for better understanding.

Coil Group

It is product of number of phases and number of poles in a rotating machine. Coil group = number of poles × the number of phases.

Balanced Winding

If under each pole face, there are an equal number of coils of different phases, then the winding is said to be balanced winding. In balanced winding, coil group should be an even number.

Unbalanced Winding

If the number of coils per coil group is not a whole number, the winding is known as unbalanced winding. In such case, each pole face contains unequal of coils of different phase. In two-phase alternator, two single-phase windings are placed on the armature by 90 electrical degrees apart from each other. In case of three phase alternator, three single-phase windings are placed on the armature, by 60 degrees (electrical) apart from each other. The figure below represents, a Skelton 2 phase 4 pole winding two slots per pole. The electrical phase difference between adjacent slots = 180/2 = 90 degree electrical).

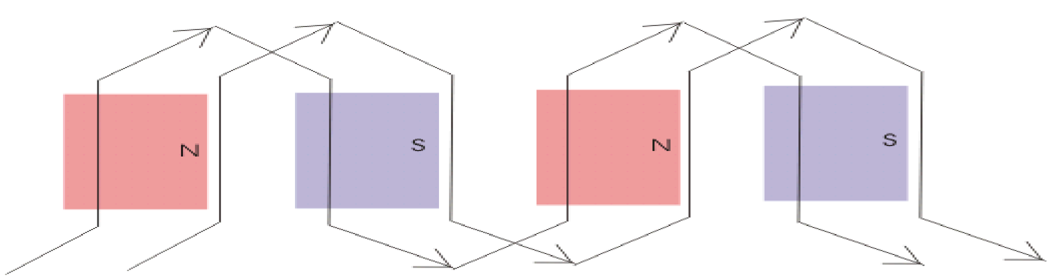


Fig - 7

Point a and b are starting point of the first and second phase winding of two, phase alternator. a' and b' are finishing point of first and second phase wining of the two-phase alternator, respectively. The figure below represents a Skelton 3 phase 4 pole winding, three slots per pole. The electrical phase difference between, adjacent slots is 180/ 3 = 60 degree (electrical) a, b and c are starting point of Red, Yellow, and blue phases and a', b', and c' are the finishing points of same Red, Yellow and Blue phases of the three-phase winding.

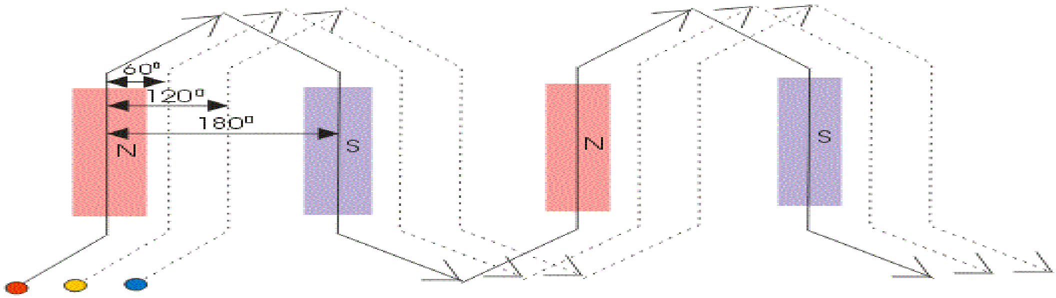


Fig - 8

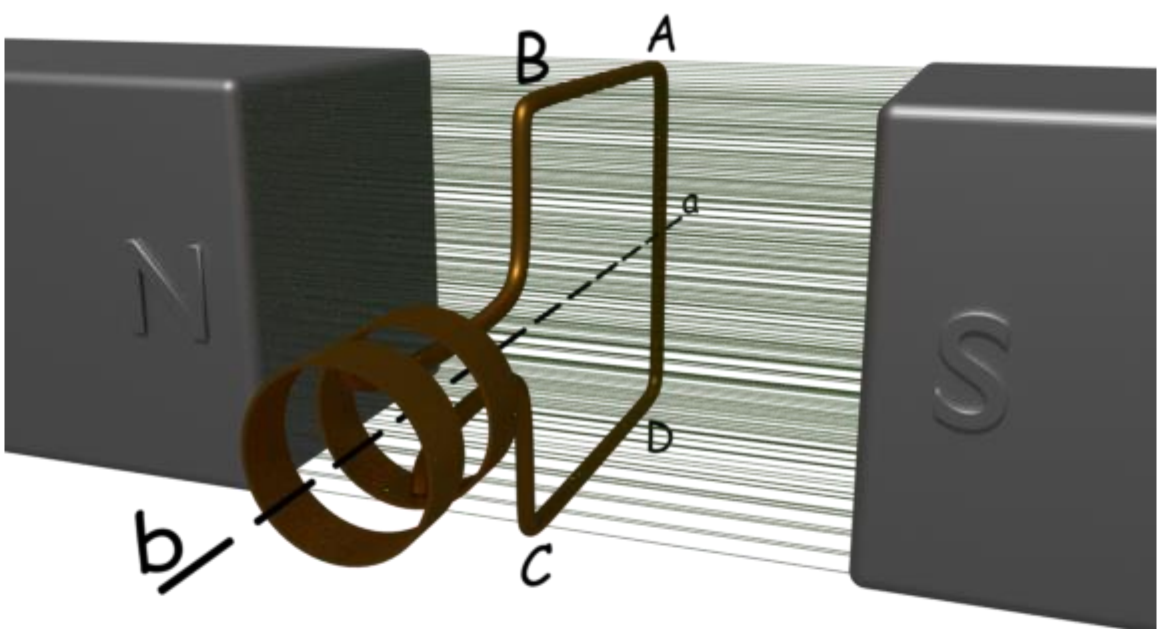
Say red phase winding starts at slot no 1 and ends over slot no 10. Then yellow winding or second winding starts at slot no 2 and ends over slot no 11. Third or blue phase winding starts at slot no 3 and ends at slot no 12. The phase difference of induced emfs, in red phase and yellow, yellow phase and blue phase and blue phase and red phase winding respectively by 60 degrees, 60 degree and 240 degrees (electrical respectively). Since in three phase system, the phase difference between red, yellow and blue phase is 120 degree (electrical). This can be achieved by revering yellow phase (second winding) winding as shown in the figure above.

Integral Slot and Fractional Slot Winding

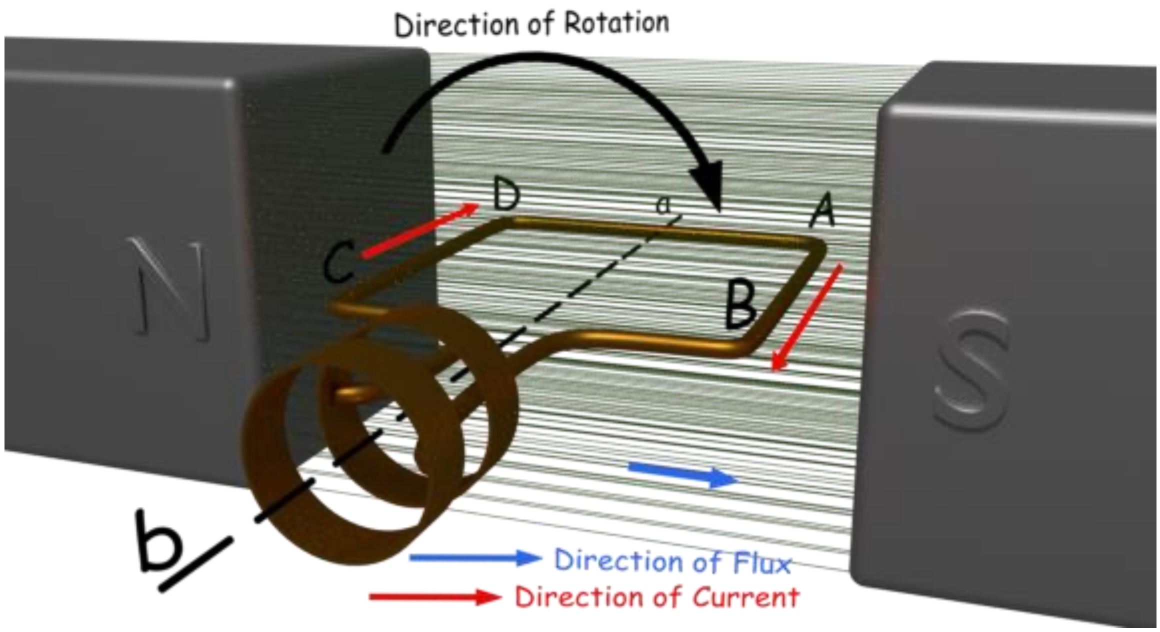
When the number of slots per pole per phase is an integer, the winding is the integer slot winding but when the number slots per pole per phase is fractional number the winding we refer as fractional slot winding. Fractional slot winding is practicable only with the double layered winding. It limits the number of parallel circuits available because phase group under several poles must be connected in series before a unit is formed and the widening respects the pattern to give the second unit that can be put in parallel with the first.

Working Principle of Alternator

The working principle of alternator is very simple. It is just like basic principle of DC generator. It also depends upon Faraday's law of electromagnetic induction which says the current is induced in the conductor inside a magnetic field when there is a relative motion between that conductor and the magnetic field.



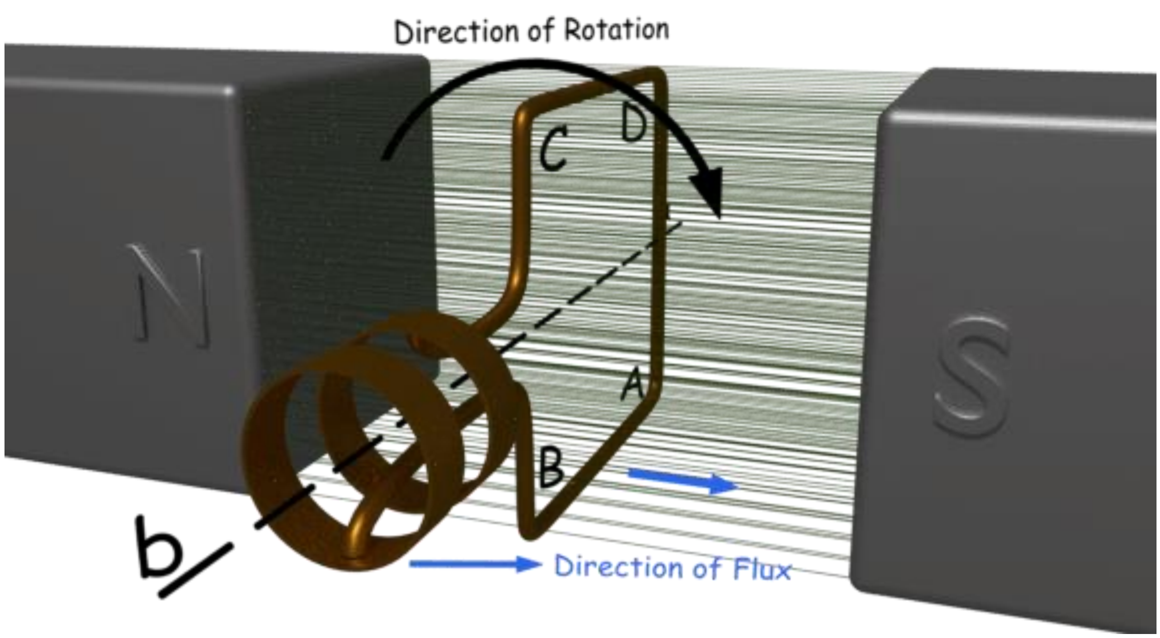
For understanding working of alternator let us think about a single rectangular turn placed in between two opposite magnetic poles as shown above.



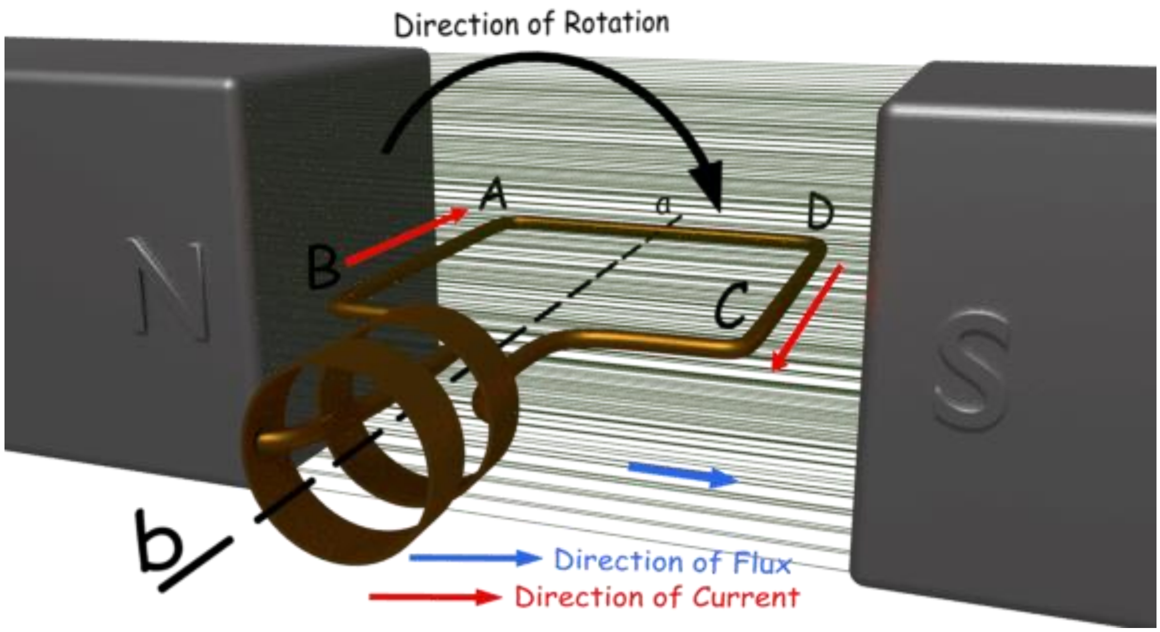
Say this single turn loop ABCD can rotate against axis a-b. Suppose this loop starts rotating clockwise. After 90^o rotation the side AB or conductor AB of the loop comes in front of S-pole and conductor CD comes in front of N-pole. At this position the tangential motion of the conductor AB is just perpendicular to the magnetic flux lines from N to S pole. Hence, the rate of flux cutting by the conductor AB is maximum here and for that flux cutting there will be an induced current in the conductor AB and the direction of the induced current can be determined by Flemming's right-hand rule. As per this rule the direction of this current will be from A to B. At the same time conductor CD comes under N pole and here also if we apply Fleming right-hand rule we will get the direction of induced current and it will be from C to D.

Now after clockwise rotation of another 90^o the turn ABCD comes at vertical position as shown below. At this position tangential motion of conductor AB and CD is just parallel to the magnetic flux lines, hence there will be no flux cutting that is no current in the conductor.

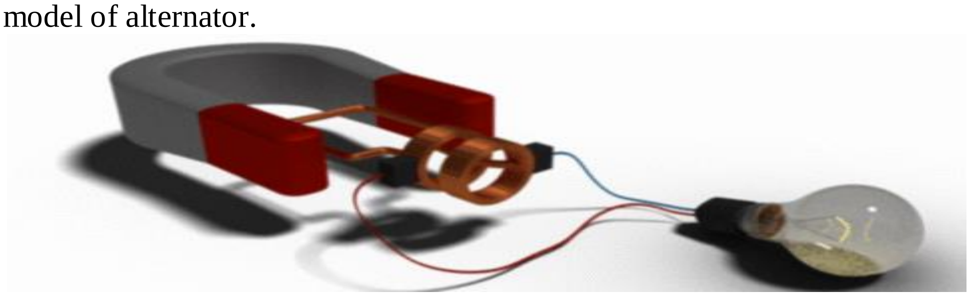
While the turn ABCD comes from horizontal position to vertical position, angle between flux lines and direction of motion of conductor, reduces from 90^o to 0^o and consequently the induced current in the turn is reduced to zero from its maximum value.



After another clockwise rotation of 90^o the turn again comes to horizontal position, and here conductor AB comes under N-pole and CD comes under S-pole, and here if we again apply Fleming right-hand rule, we will see that induced current in conductor AB, is from point B to A and induced current in the conductor CD is from D to C.

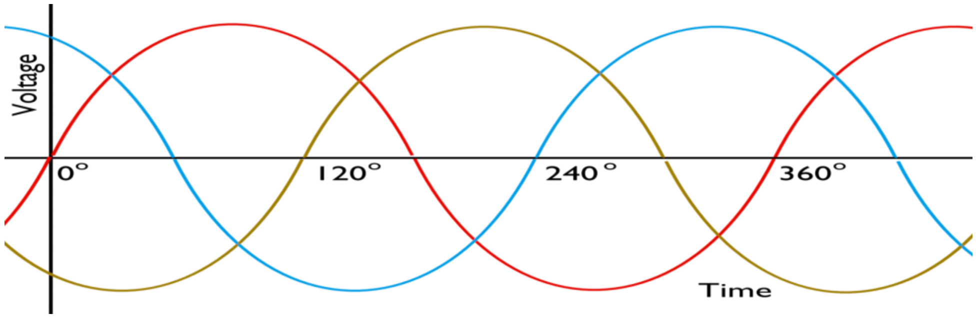


As at this position the turn comes at horizontal position from its vertical position, the current in the conductors comes to its maximum value from zero. That means current is circulating in the close turn from point B to A, from A to D, from D to C and from C to B, provided the loop is closed although it is not shown here. That means the current is in reverse of that of the previous horizontal position when the current was circulating as A → B → C → D → A.
While the turn further proceeds to its vertical position the current is again reduced to zero. So if the turn continues to rotate the current in turn continually alternate its direction. During every full revolution of the turn, the current in turn gradually reaches to its maximum value then reduces to zero and then again it comes to its maximum value but in opposite direction and again it comes to zero. In this way, the current completes one full sine wave cycle during each 360° revolution of the turn. So, we have seen how an alternating current is produced in a turn is rotated inside a magnetic field. From this, we will now come to the actual working principle of alternator.
Now we place one stationary brush on each slip ring. If we connect two terminals of an external load with these two brushes, we will get an alternating current in the load. This is our elementary model of alternator.



Having understood the very basic principle of an alternator, let us now have an insight into its basic operational principle of a practical alternator. During discussion of basic working of alternator, we have considered that the magnetic field is stationary and conductors (armature) is rotating. But generally in practical construction of alternator, armature conductors are stationary and field magnets rotate between them. The rotor of an alternator or a synchronous generator is mechanically coupled to the shaft or the turbine blades, which being made to rotate at synchronous speed N_s under some mechanical force results in magnetic flux cutting of the stationary armature conductors housed on the stator. As a direct consequence of this flux cutting an induced emf and current starts to flow through the armature conductors which first flow in one direction for the first half cycle and then in the other direction for the second half cycle for each winding with a definite time lag of 120° due to the space displaced arrangement of 120°

between them as shown in the figure below. This particular phenomenon results in three phase power flow out of the alternator which is then transmitted to the distribution stations for domestic and industrial uses.



Winding Factor | Pitch Factor | Distribution Factor

Before knowing about, winding factor, we should know about pitch factor and distribution factor, since winding factor is the product of pitch factor and distribution factor.If we denote winding factor with K_w, pitch factor with K_p and distribution factor with K_d, we can write

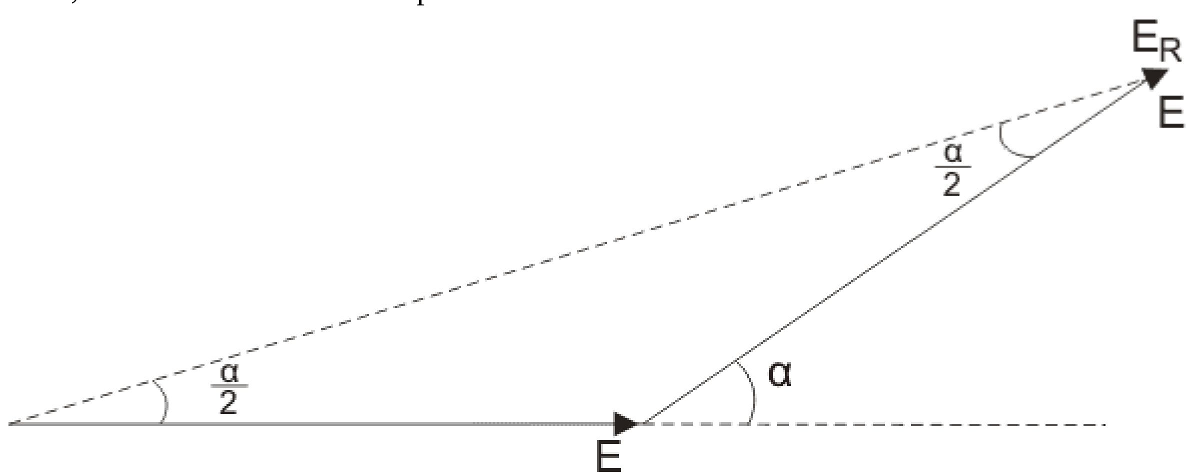
$$K_w = K_p \times K_d$$

The pitch factor and distribution factor are explained below one by one.

Pitch Factor

In short pitched coil, the induced emf of two coil sides get vectorially added and give resultant emf of the loop. In short pitched coil, the phase angle between the induced emf of two opposite coil sides is less than 180^o (electrical). But we know that, in full pitched coil, the phase angle between the induced emf of two coil sides is exactly 180^o (electrical). Hence, the resultant emf of a full pitched coil is just the arithmetic sum of the emfs induced on both sides of the loop. We well know that vector sum or phasor sum of two quantities is always less than their arithmetic sum. The pitch factor is the measure of resultant emf of a short-pitched coil in comparison with resultant emf of a full pitched coil.

Hence, it must be the ratio of phasor sum of induced emfs per coil to the arithmetic sum of induced emfs per coil. Therefore, it must be less than unity. Let us assume that, a coil is short pitched by an angle α (electrical degree). Emf induced per coil side is E. The arithmetic sum of induced emfs is 2E. That means, 2E, is the induced voltage across the coil terminals, if the coil would have been full pitched. Now, come to the short pitched coil. From the figure below it is clear that, resultant emf of the short pitched coil



$$E_R = 2E\cos\frac{\alpha}{2}$$

Now, as per definition of pitched factor,

$$K_p = \frac{Resultant\ emf\ of\ short\ pitched\ coil}{Resultant\ emf\ of\ full\ pitched\ coil}$$
$$= \frac{Phasor\ sum\ of\ coil\ side\ emfs}{Arithmetic\ sum\ of\ coil\ side\ emfs}$$
$$= \frac{2E\cos\frac{\alpha}{2}}{2E} = \cos\frac{\alpha}{2}$$

This pitch factor is the fundamental component of emf. The flux wave may consist of space field harmonics also, which give rise to the corresponding time harmonics in the generated voltage waveform. A 3rd harmonic component of the flux wave, may be imagined as produced by three poles as compared to one pole for the fundamental component. In the view of this, the chording angle for the rth harmonic becomes r times the chording angle for the fundamental component and pitch factor for the rth harmonic is

$$cos\frac{r\alpha}{2} = 0\ or\ \frac{r\alpha}{2} = 90^o$$

$$K_{pr} = cos\frac{r\alpha}{2}$$

given as, The rth harmonic becomes zero, if, In 3 phase alternator, the 3rd harmonic is suppressed by star or delta connection as in the case of 3 phase transformer. Total attention is given for designing a 3 phase alternator winding design, for 5th and 7th harmonics. For 5th harmonic

$$\frac{5\alpha}{2} = 90^\circ \Rightarrow \alpha = \frac{180^\circ}{5} = 36^\circ$$
$$\frac{7\alpha}{2} = 90^\circ \Rightarrow \alpha = \frac{180^\circ}{7} = 25.7^\circ$$

For 7th harmonic Hence, by adopting a suitable chording angle of α = 30°, we make most optimized design armature winding of alternator.

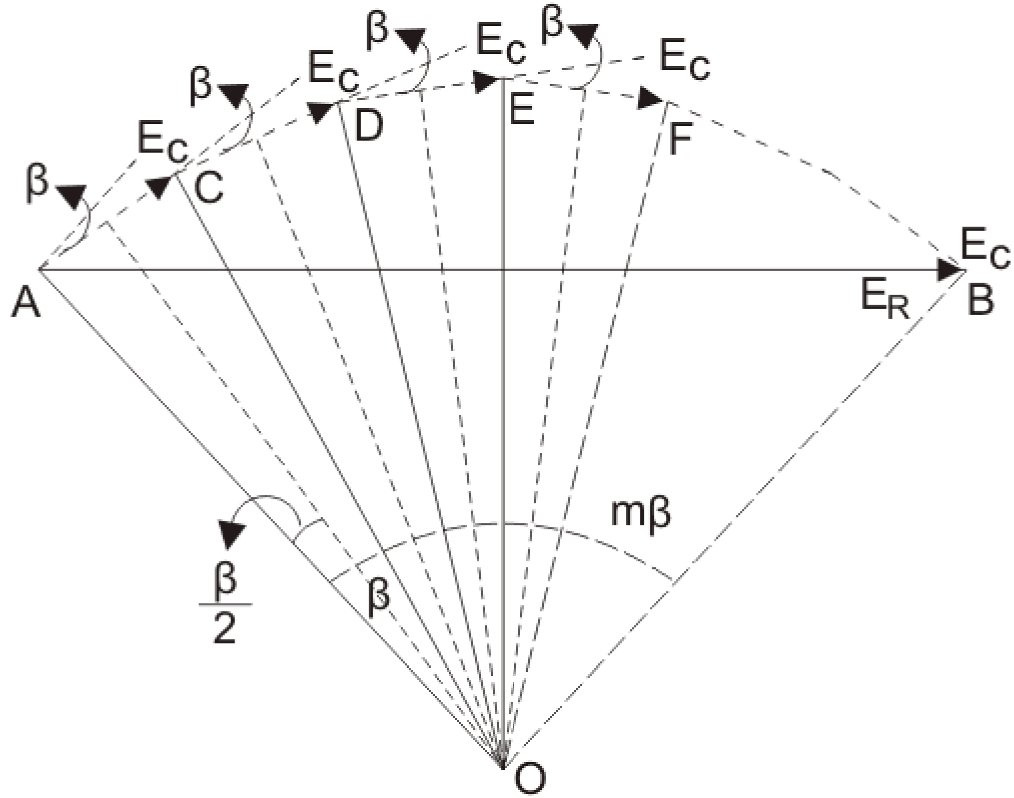
Distribution Factor

If all the coil sides of any one phase under one pole are bunched in one slot, the winding obtained is known as concentrated winding and the total emf induced is equal to the arithmetic sum of the emfs induced in all the coils of one phase under one pole. But in practical cases, for obtaining smooth sinusoidal voltage waveform, armature winding of alternator is not concentrated but distributed among the different slots to form polar groups under each pole. In distributed winding, coil sides per phase are displaced from each other by an angle equal to the angular displacement of the adjacent slots. Hence, the induced emf per coil side is not an angle equal to the angular displacement of the slots. So, the resultant emf of the winding is the phasor sum of the induced emf per coil side. As it is phasor sum, must be less than the arithmetic sum of these induced emfs. Resultant emf would be an arithmetic sum if the winding would have been a concentrated one. As per definition, distribution factor is a measure of resultant emf of a distributed winding in compared to a concentrated winding.

We express it as the ratio of the phasor sum of the emfs induced in all the coils distributed in some slots under one pole to the arithmetic sum of the emfs induced. Distribution factor is,

$$k_d = \frac{EMF\ induced\ in\ distributed\ winding}{EMF\ induced\ if\ the\ winding\ would\ have\ been\ concentrated}$$

$$= \frac{Phasor\ sum\ of\ component\ emfs}{Arithmetic\ sum\ of\ component\ emfs}$$



As pitch factor, distribution factor is also always less than unity. Let the number of slots per pole is n. The number of slots per pole per phase is m. Induced emf per coil side is E_c. Angular

$$\beta = \frac{180^o}{n}$$

displacement between the slots, Let us represent the emfs induced in different coils of one phase under one pole as AC, DC, DE, EF and so on. They are equal in magnitude, but they differ from each other by an angle β. If we draw bisectors on AC, CD, DE, EF --------. They would meet at

$$E = AC = 2 \cdot OAsin\frac{\beta}{2}$$

common point O. Emf induced in each coil side,

As the slot per pole per phase is m, the total arithmetic sum of all induced emfs per coil

$$Arithmetic\ sum = m \times 2 \times OAsin\frac{\beta}{2}$$

sides per pole per phase, The resultant emf would be AB, as represented by the figure,

$$E_R = AB = 2 \times OA \sin \frac{\angle AOB}{2} = 2 \times OA \sin \frac{m\beta}{2}$$

$$Therefore,\ Distribution\ Factor$$

$$K_d = \frac{Phasor\ sum\ of\ component\ emfs}{Arithmetic\ sum\ of\ component\ emfs}$$

$$= \frac{2 \times OA \sin \frac{m\beta}{2}}{m \times 2 \times OA \sin \frac{\beta}{2}} = \frac{\sin \frac{m\beta}{2}}{m \sin \frac{\beta}{2}}$$

Hence, the resultant emf mβ is also known as the phase spread in electrical degree. The distribution factor K_d given by equation is for the fundamental component of emf. If the flux distribution contains space harmonics the slot angular pitch β on the fundamental scale, would become rβ for the r^{th} harmonic component and thus the distribution factor

$$K_{dr} = \frac{\sin \frac{rm\beta}{2}}{m \sin \frac{r\beta}{2}}$$

$$Therefore,\ Winding\ Factor$$

$$K_w = K_p \times K_d = \cos \frac{\alpha}{2} \times \frac{\sin \frac{m\beta}{2}}{m \sin \frac{\beta}{2}}$$

for the r^{th} harmonic would be.

EMF Equation of an Alternator and AC Generator

An alternator or AC generator (dynamo) is a device which convert mechanical energy to electrical energy. When we supply the magnetizing current by DC shunt generator through two slip rings (in recent alternators, they use electronic starting system instead of slip rings and commutators) because the field magnets are rotating. keep in mind that most alternators use a rotating magnetic field with a stationary armature.

When the rotor rotates, the stator conductors which are static in case of alternator cut by magnetic flux , they have induced EMF produced in them (according to Faraday's law of electromagnetic induction which states that if a conductor or coil links with any changing flux, there must be an induced emf in it.

This induced EMF can be found by the EMF equation of the alternator which as follow:

Lets,

P = No. of poles

Z = No. of Conductors or Coil sides in series/phase i.e. Z = 2T…Where T is the number of coils or turns per phase (Note that one turn or coil has two ends or sides)

f = frequency of induced EMF in Hz

Φ = Flux per pole (Weber)

N = rotor speed (RPM)

K_d= Distribution factor = $\frac{\text{Sin } m\,\beta/2}{m \text{ Sin } \beta/2}$

Where Distribution factor = K_d = $\frac{\text{e.m.f. with distributed winding}}{\text{e.m.f. with concentrated winding}}$

K_c or K_P = Cos α/2

If induced EMF is assumed sinusoidal then,

K_f = Form factor = 1.11

In one revolution of the rotor i.e. in 60/N seconds, each conductor is cut by a flux of ΦP Webers.

$d\Phi = \Phi P$ and also $d\Phi$ = 60/N seconds

then induced e.m.f per conductor (average) = $= \frac{d\Phi}{dt} = \frac{\Phi P}{60/N} = \frac{\Phi NP}{60}$ ….. *(i)*

But we know that:

f = PN / 120 or N= 120f / P

Putting the value of N in Equation *(i)*, we get,

Average value of EMF per conductor = $= \frac{\Phi P}{60} \times \frac{120 f}{P} = 2 f\ \Phi$ volt

∴ (N= 120f/P)

If there are Z conductors in series per phase,

then average e.m.f per phase = 2 f Φ Z Volts = 4 f ΦT *Volts* ….. (Z=2T)

Also we know that;

Form Factor= RMS Value / Average Value

= RMS value= Form factor x Average Value,

= 1.11 x 4fΦT = 4.44fΦT Volts.

(Note that is exactly the same equation as the EMF equation of the transformer)

And the actual available voltage per phase

= 4 K_c K_d f ΦT = 4 K_f K_c K_d f ΦT Volts.

Note: If alternator or AC generator is star connected as usually the case, then the Line Voltage is √3 times the phase voltage as derived from the the above formula.

ARMATURE REACTION:

Armature winding in an electrical machine is the winding which carries the load current. Under no-load condition, the armature current is zero. But as the machine is loaded, load current flows through the armature winding and creates magnetic flux. The effect of armature winding mmf or flux on the main working flux created by field poles is called the armature reaction. This article outlines the armature reaction in synchronous machine or alternator.

To better understand the armature reaction in synchronous machine, it is essential to first understand the internal happening. For this purpose the alternator operation is considered under different operating power factor and loads as follows:

- No-load Operation
- Unity power factor (pf) load
- Zero power factor lagging load
- Zero power factor leading load
- Lagging power factor load

Let us now investigate the effect of armature reaction for the above mentioned cases one by one.

No-load Operation:

As mentioned earlier, the current through the armature winding of alternator is zero for no-load operation; therefore there will not be any armature reaction. When the field winding is excited by a DC source and the alternator / generator is brought up to synchronous speed by adjusting the speed of prime-mover, no load voltage or excitation voltage is generated across the alternator armature terminals. The value of no-load or excitation armature terminal voltage is given as

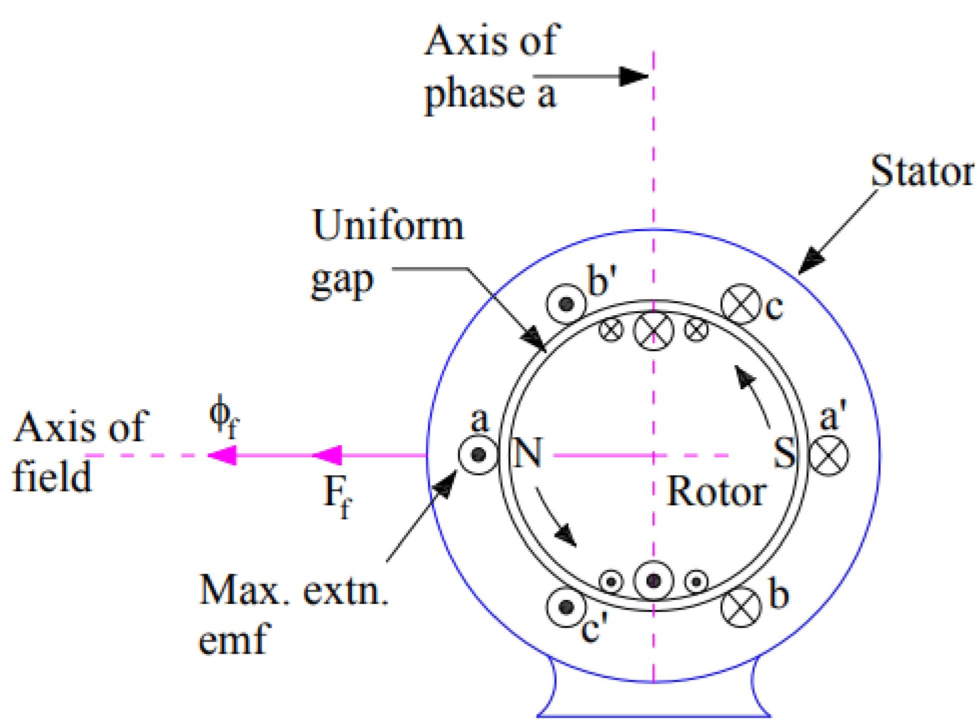
$E_f = \sqrt{2}\pi f N_{ph} K_w Ø_f$

Where E_f, N_{ph}, K_w and $Ø_f$ are excitation voltage, number of series turns per phase, winding factor and filed flux respectively.

This generated excitation voltage across the armature terminal lags behind the field flux $Ø_f$ by 90 degree. In general, the emf generated by the filed flux always lags behind the field flux by 90 degree in any machine.

Unity Power Factor Load:

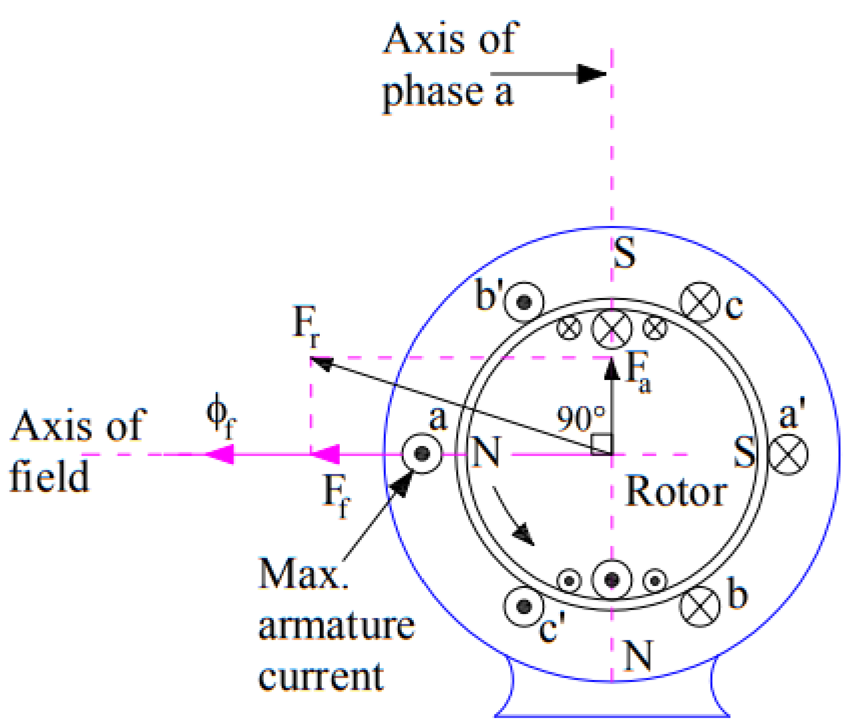
In order to understand the effect of armature mmf on the field mmf wave, let us consider a two pole cylindrical rotor alternator as shown in figure below.



In the above figure, concentrated full pitched coils aa',bb' and cc' on stator represents the three phase winding a, b & c. The filed winding on rotor is fed by DC source for setting up working or main filed mmf. Field current indicated by cross and dot in the field winding on rotor, creates field mmf Ff and field flux $Ø_f$ which are sinusoidally distributed along the air-gap periphery. This filed flux creates North (N) and South (S) pole on the rotor.

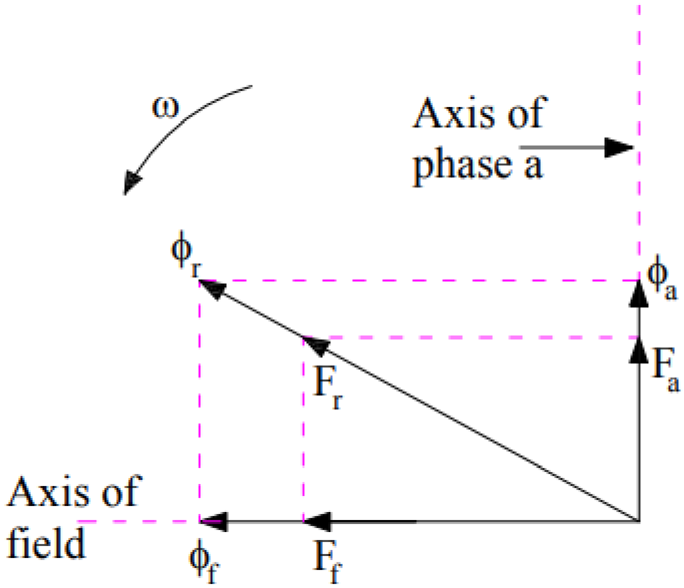
As we know that emf induced in a coil is maximum when its coil sides are lying under maximum flux position. In view of this, maximum emf will be generated in coil aa' as its coil sides a and a' are lying under peak flux density. This generated emf is shown by dot and cross at a & a' respectively assuming anti-clockwise rotation of rotor. Since coil sides b' and c' are also under the influence of N pole, emf induced in these coil are represented by a dot. However their magnitude will be less than the maximum value. This emf generated by $Ø_f$ alone is called the excitation voltage.

Now, when this alternator is connected to a balanced 3 phase load, a balanced three phase current starts flowing in the three phases of alternator. As the load is of unity power factor, this means that excitation voltage E_f and armature current I_a will be in phase. This can also be interpreted in another way like excitation voltage and armature current attain their peak simultaneously. Since excitation voltage is maximum in phase "a", this means armature current phase a will also be maximum. Though load current also flows in remaining two phases "b" and "c" but their magnitude is less than the maximum for this instant of time. *The mmf set up by the armature current is called the armature reaction mmf*. As we know that for balanced polyphase currents flowing in the polyphase winding, the peak value of resultant mmf wave is along that phase axis which carries the maximum current. Therefore, the resultant armature reaction mmf F_a due to the combined effect 3-phase mmfs is set up along the phase "a" because this phase carries the maximum current. This is shown in figure below.



This armature reaction mmf wave produces North (N) and South (S) pole on the stator as shown in above figure. The interaction between these poles on stator and rotor causes the production of electromagnetic torque. For aa alternator, the prime-mover torque must counteracts this electromagnetic torque for conversion of mechanical energy into electrical energy.

Now, if we combine the space phasor of field mmf and armature mmf wave, then it can easily be seen that the armature reaction mmf lags behind the filed mmf by 90 degree. This is shown in figure below.



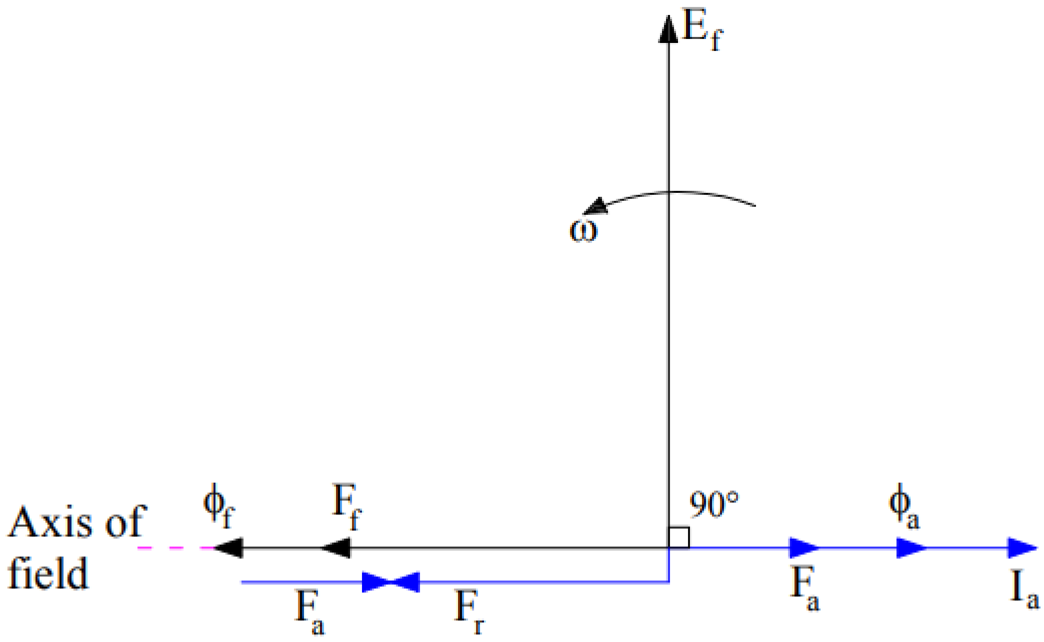
The resultant air gap mmf will be the resultant of filed mmf and armature reaction mmf. This means,

$F_r = F_f + F_a$

If we neglect the saturation, then the field flux $Ø_f$ and armature flux $Ø_a$ will be along their respective mmf wave. This is shown in the figure. Thus we can say that, armature reaction flux lags behind the field flux by 90 degree. Therefore, *armature reaction mmf at unity power factor is entirely cross-magnetizing in nature.*

Zero Power Factor Lagging Load:

Zero power factor lagging load means that the load current is lagging behind the excitation voltage by 90 degree. This is shown in figure below.



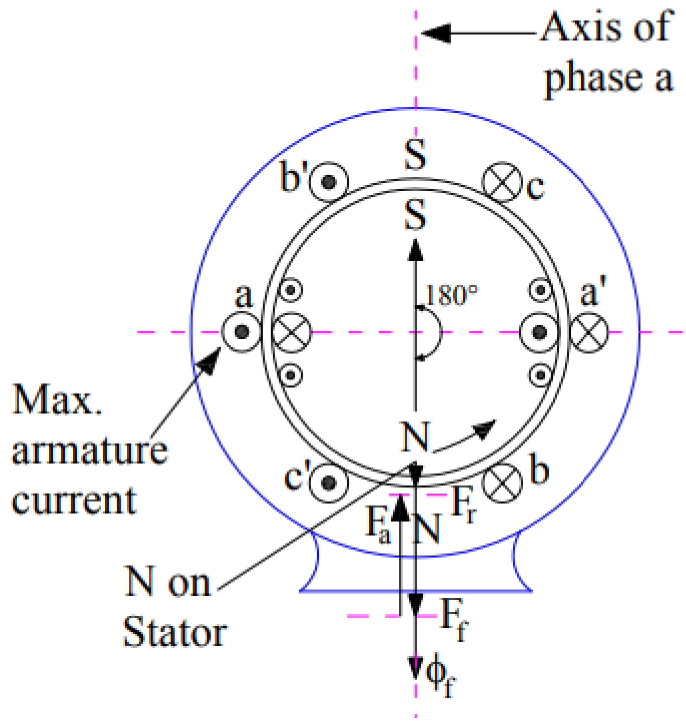
The above phasor diagram has been drawn using the following facts:

- The excitation emf lags the field mmf by 90 degree.
- As saturation is neglected, the filed flux will be along the field mmf.
- Armature reaction mmf is along the armature current.

From the above phasor diagram, it is clear that armature reaction mmf F_a is in opposition of field mmf $Ø_f$. This means that the resultant air-gap mmf will be equal to ($F_f - F_a$). *Thus under zero power factor lagging loading condition of alternator, the effect of armature reaction mmf is purely demagnetizing.*

This can also be understood in another way as described below.

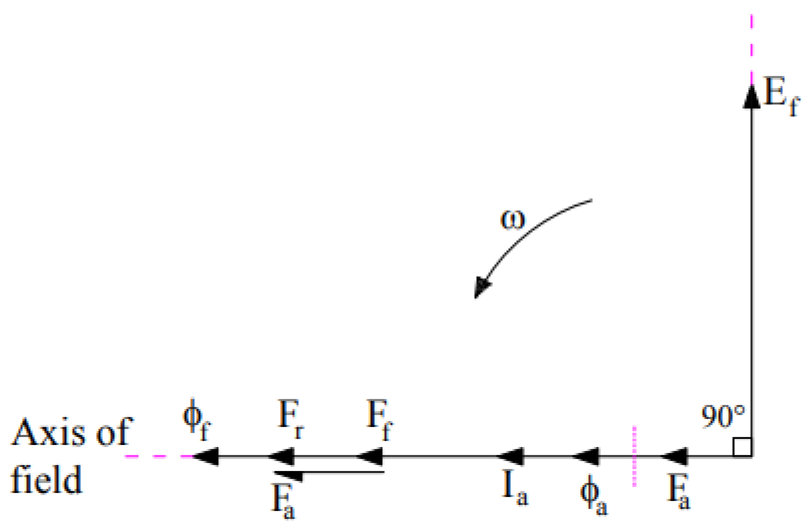
Due to zero power factor lagging load, the current in phase "a" of armature winding will become maximum when the field poles have advanced by 90 degree in assumed counter-clockwise direction. This is shown in figure below.



Carefully observe the above figure. It can be seen that, in this case the direction of armature reaction mmf F_a will be along the phase "a" axis and that of and filed mmf $Ø_f$ will be vertically downward. This direction can be found by using right hand screw rule. Thus we see that, both the armature reaction and filed mmf are opposing each other. Hence, the effect of armature reaction mmf is purely demagnetizing in zero pf lagging load condition.

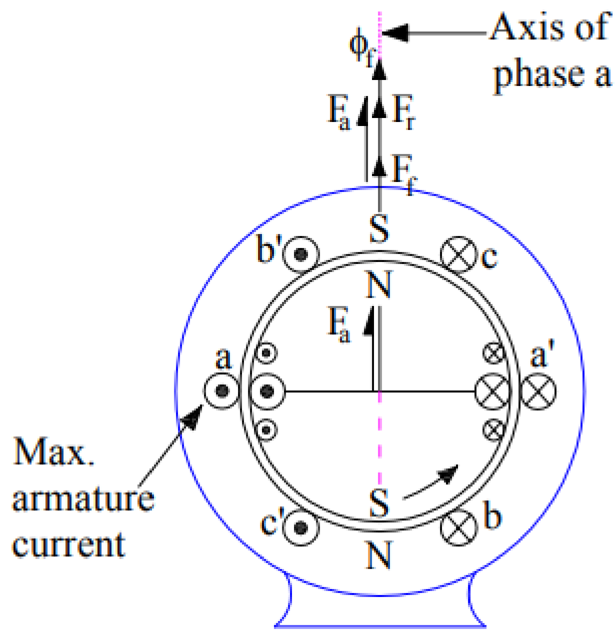
Zero Power Factor Leading Load:

The condition of zero pf leading load may be depicted by phasor diagram in the same way that for zero pf lagging load. This is shown below.



From the above phasor, it is clear that armature reaction mmf is in the direction of field mmf. Thus the effect of armature reaction mmf under zero power factor leading loads is purely magnetizing.

The above conclusion can also be drawn by analyzing the space phasor of the alternator. Zero pf leading means that the current in phase “a” of armature winding will be zero when the field poles are behind the axis of phase “a“ winding by 90 degree as shown in figure below.

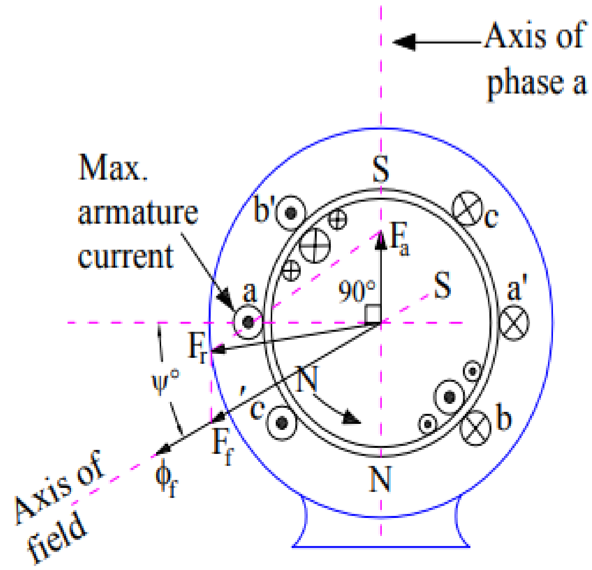


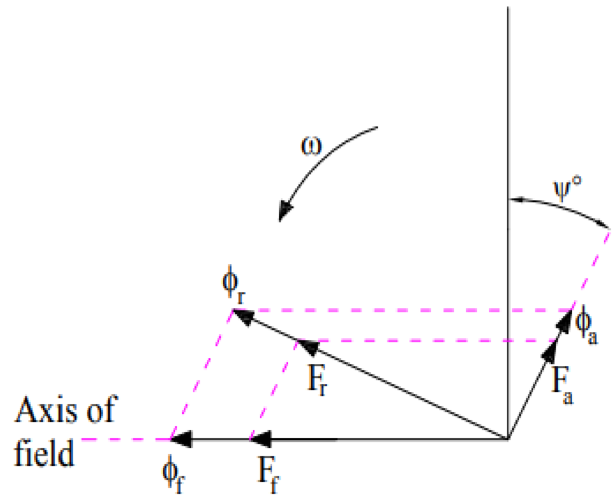
Therefore, the direction of field mmf and armature mmf will be same. Hence armature reaction mmf is magnetizing in nature.

Lagging Power Factor Load:

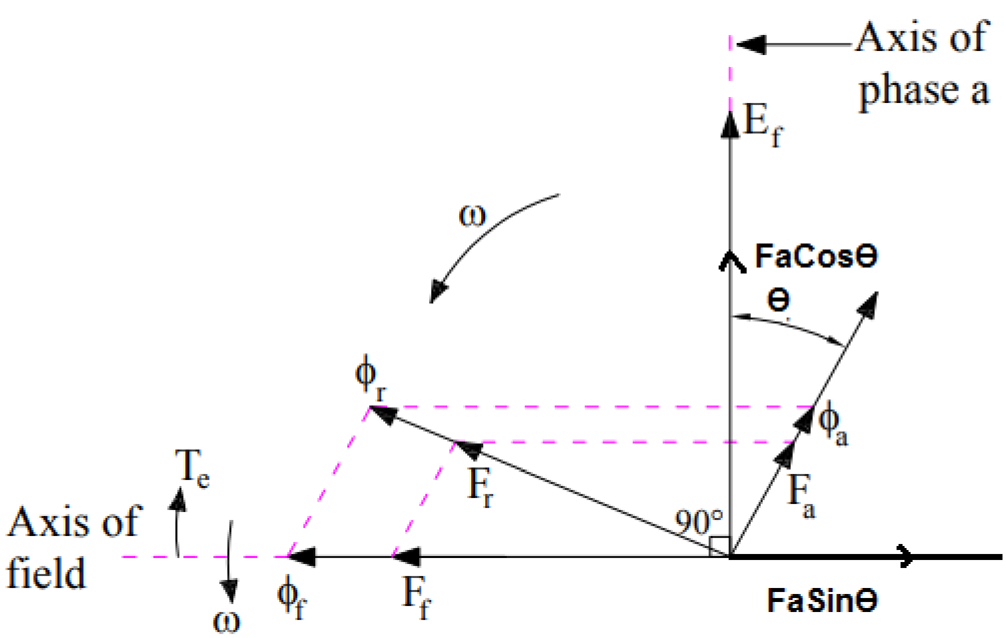
Lagging power factor loads are more common. So let us consider a general case of armature current I_a lagging the excitation voltage E_f by angle ϴ. This means that the load power factor is cosθ.

For a lagging pf load having pf angle θ, the current in coil a and a’ will be maximum when the field poles have advance by θ degree in space. In other words, by the time armature current in coil a, a’ attains maximum value with the same polarity i.e. dot in coil side “a”, the rotor poles N, S would have moved forward by θ degree as shown in figure below.





The resultant of rotating armature mmf F_a is directed vertically upward along the axis of phase "a", because this phase carries the maximum current at the instant considered. Thus the armature reaction mmf F_a is lagging behind the filed mmf $Ø_f$ by (90+θ) degree. Let us now draw the phasor diagram.



The armature reaction mmf can be resolved into two components: One along the excitation voltage E_f (F_aCosθ) and another opposite to the field mmf F_f (F_aSinθ). Thus we can say that, the effect of armature reaction in lagging pf load is cross-magnetizing as well as demagnetizing in nature.

Thus to summarize, the effect of armature reaction mmf on main filed mmf of alternator is tabulated below.

Sr. No.	Loading Condition	Effect of Armature Reaction
1)	No Load	No effect
2)	Unity Power Factor	Cross-magnetizing
3)	Zero Power Factor Lagging	Purely demagnetizing
4)	Zero Power Factor Leading	Purely magnetizing
5)	Lagging Load	Cross-magnetizing and demagnetizing

Voltage Regulation of a Synchronous Generator

The Voltage Regulation of a Synchronous Generator is the rise in voltage at the terminals when the load is reduced from full load rated value to zero, speed and field current remaining constant. It depends upon the power factor of the load. For unity and lagging power factors, there is always a voltage drop with the increase of load, but for a certain leading power, the full load voltage regulation is zero.

The voltage regulation is given by the equation shown below.

$$\text{Per Unit Voltage Regulation} \triangleq \frac{|E_a| - |V|}{|V|} \quad \ldots\ldots\ldots(1)$$

$$\text{Percentage Voltage Regulation} \triangleq \frac{|E_a| - |V|}{|V|} \quad \ldots\ldots\ldots(2)$$

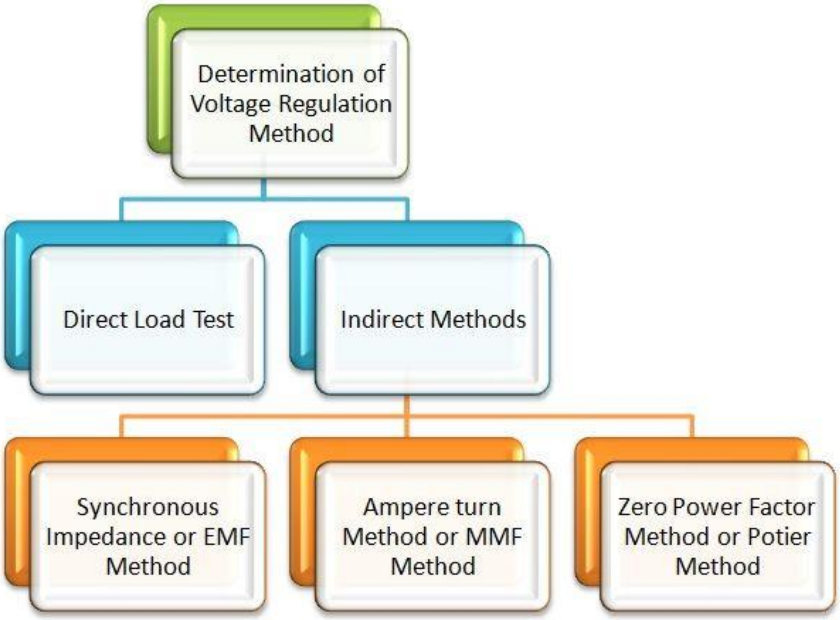
Where,

- $|E_a|$ is the magnitude of a generated voltage per phase
- $|V|$ is the magnitude of rated terminal voltage per phase

In this case, the terminal voltage is the same for both full load and no load conditions. At lower leading power factors, the voltage rises with the increase of load, and the regulation is negative.

Determination of Voltage Regulation

There are mainly two methods which are used to determine the regulation of voltage of a smooth cylindrical rotor type alternators. They are named as direct load test method and indirect methods of voltage regulation. The indirect method is further classified as Synchronous Impedance Method, Ampere-turn Method and Zero Power Factor Method.



Direct Load Test

The alternator runs at synchronous speed, and its terminal voltage is adjusted to its rated value V. The load is varied until the Ammeter and Wattmeter indicate the rated values at the given power factor. The load is removed, and the speed and the field excitation are kept constant. The value of the open circuit and no load voltage is recorded.

It is also found from the percentage voltage regulation and is given by the equation shown below.

$$\% \text{ Voltage Regulation} = \frac{E_a - V}{V} \times 100\%$$

The method of direct loading is suitable only for small alternators of the power rating less than 5 kVA.

Indirect Methods of Voltage Regulation

For large alternators, the three indirect methods are used to determine the voltage regulation they are as follows.

Synchronous Impedance Method

The Synchronous Impedance Method or Emf Method is based on the concept of replacing the effect of armature reaction by an imaginary reactance. For calculating the regulation, the synchronous method requires the following data; they are the armature resistance per phase and the open circuit characteristic.The open circuit characteristic is the graph of the circuit voltage and the field current. This method also requires short circuit characteristic which is the graph of the short circuit and the field current.

Contents:

- DC resistance test

- Open Circuit Test
- Short Circuit Test
- Calculation of Synchronous Impedance
- Assumptions in the Synchronous Impedance Method

For a synchronous generator following are the equation given below

$$V = E_a - Z_s I_a$$

Where,

$$Z_s = R_a + jX_s$$

For calculating the synchronous impedance, Z_s is measured, and then the value of E_a is calculated. From the values of E_a and V, the voltage regulation is calculated.

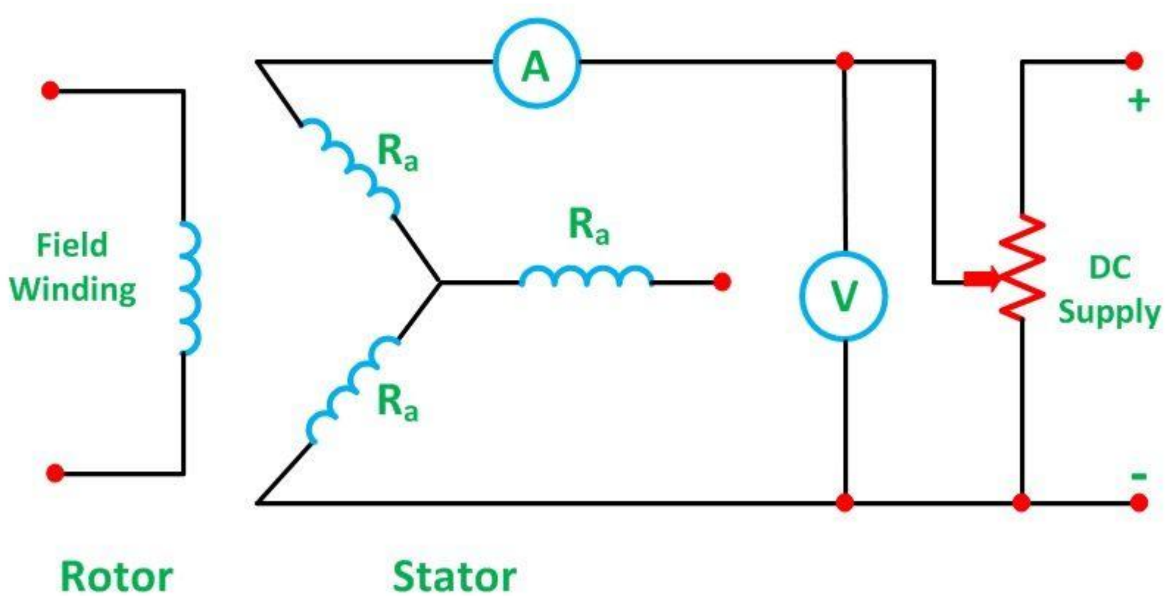
Measurement of Synchronous Impedance

The measurement of synchronous impedance is done by the following methods. They are known as

- DC resistance test
- Open circuit test
- Short circuit test

DC resistance test

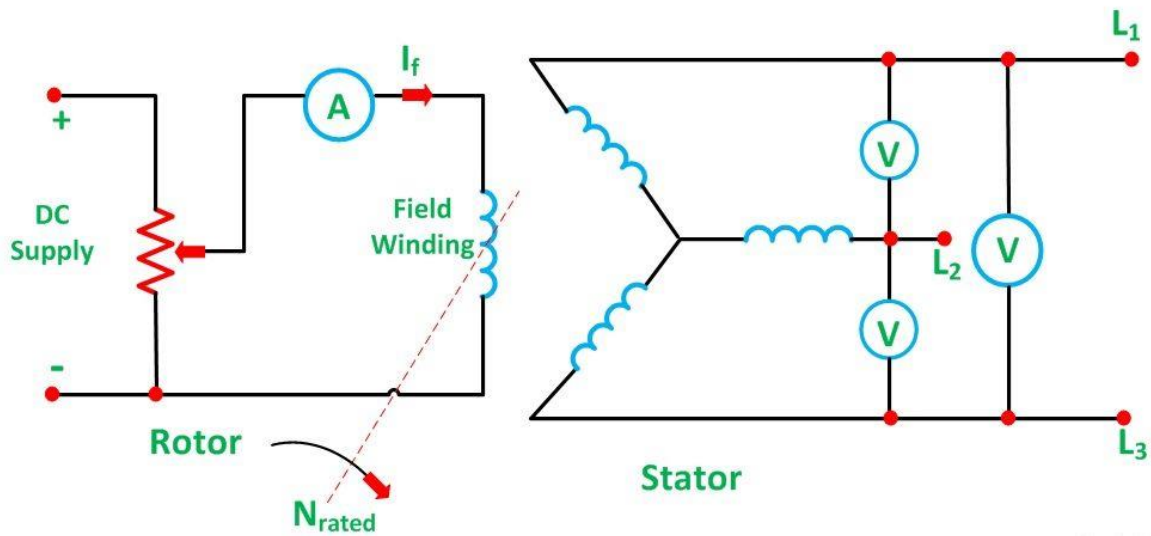
In this test, it is assumed that the alternator is star connected with the DC field winding open as shown in the circuit diagram below.



It measures the DC resistance between each pair of terminals either by using an ammeter – voltmeter method or by using the Wheatstone's bridge. The average of three sets of resistance value R_t is taken. The value of R_t is divided by 2 to obtain a value of DC resistance per phase. Since the effective AC resistance is larger than the DC resistance due to skin effect. Therefore, the effective AC resistance per phase is obtained by multiplying the DC resistance by a factor 1.20 to 1.75 depending on the size of the machine. A typical value to use in the calculation would be 1.25.

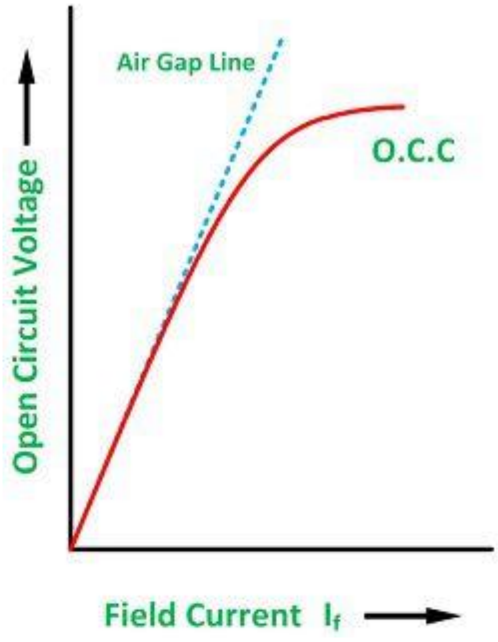
Open Circuit Test

In the open circuit test for determining the synchronous impedance, the alternator is running at the rated synchronous speed, and the load terminals are kept open. This means that the loads are disconnected, and the field current is set to zero. The circuit diagram is shown below.



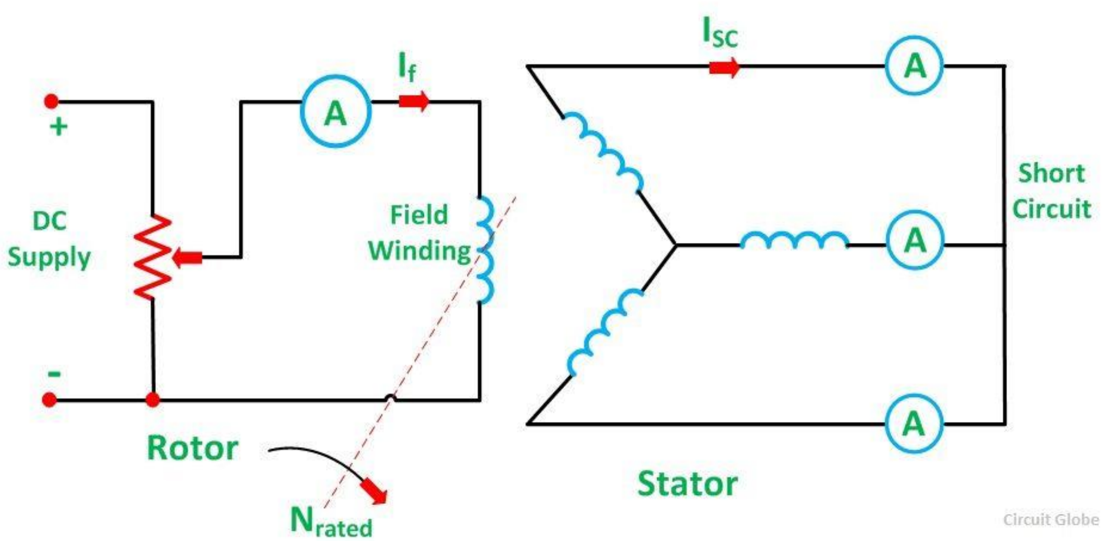
After setting the field current to zero, the field current is gradually increased step by step. The terminal voltage E_t is measured at each step. The excitation current may be increased to get 25% more than the rated voltage. A graph is drawn between the open circuit phase voltage $E_p = E_t/\sqrt{3}$ and the field current I_f. The curve so obtains called Open Circuit Characteristic (O.C.C). The shape is same as normal magnetisation curve. The linear portion of the O.C.C is extended to form an air gap line.

The Open Circuit Characteristic (O.C.C) and the air gap line is shown in the figure below.



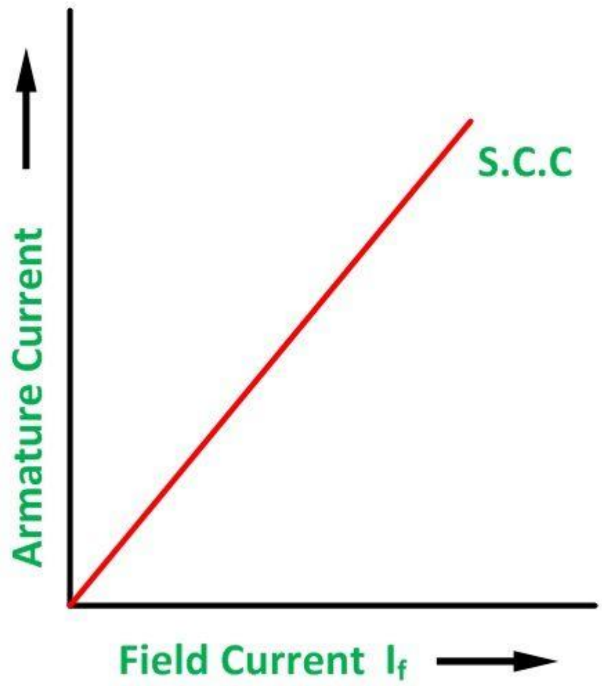
Short Circuit Test

In the short circuit test, the armature terminals are shorted through three ammeters as shown in the figure below.



The field current should first be decreased to zero before starting the alternator. Each ammeter should have a range greater than the rated full load value. The alternator is then run at synchronous speed. Same as in an open circuit test that the field current is increased gradually in steps and the armature current is measured at each step. The field current is increased to get armature currents up to 150% of the rated value.

The value of field current If and the average of three ammeter readings at each step is taken. A graph is plotted between the armature current Ia and the field current If. The characteristic so obtained is called Short Circuit Characteristic (S.C.C). This characteristic is a straight line as shown in the figure below.



Calculation of Synchronous Impedance

The following steps are given below for the calculation of the synchronous impedance.

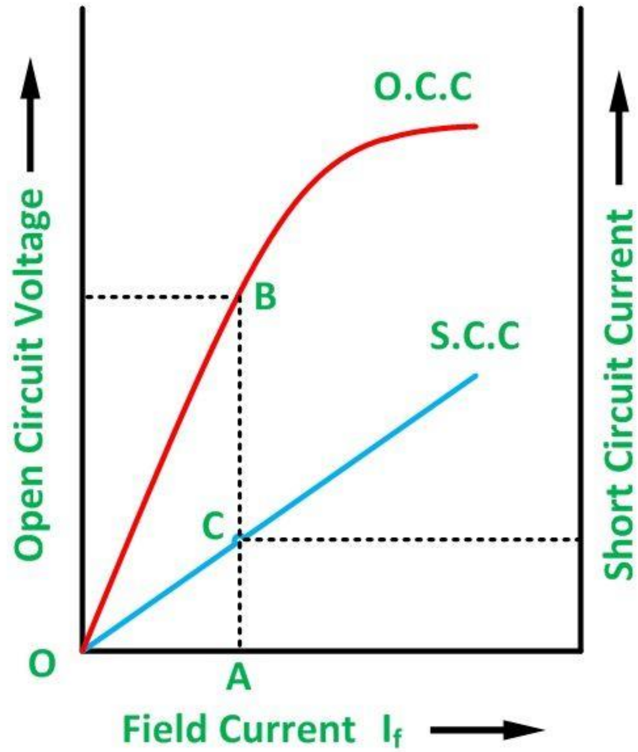
- The open circuit characteristics and the short circuit characteristic are drawn on the same curve.
- Determine the value of short circuit current I_{sc} and gives the rated alternator voltage per phase.
- The synchronous impedance Z_S will then be equal to the open circuit voltage divided by the short circuit current at that field current which gives the rated EMF per phase.

$$Z_S = \frac{\text{Open circuit voltgae per phase}}{\text{Short circuit armature current}} \quad \text{(for the same value of field current)}$$

The synchronous reactance is determined as

$$X_S = \sqrt{Z_S^2 - R_a^2}$$

The graph is shown below.



From the above figure consider the field current I_f = OA that produces rated alternator voltage per phase. Corresponding to this field current, the open circuit voltage is AB

Therefore,

$$Z_S = \frac{AB \text{ (in volts)}}{AC \text{ (in amperes)}}$$

Assumptions in the Synchronous Impedance Method

The following assumptions made in the synchronous Impedance Method are given below.

- The synchronous Impedance is constant

The synchronous impedance is determined from the O.C.C and S.C.C. It is the ratio of the open circuit voltage to the short circuit current. When the O.C.C and S.C.C are linear, the synchronous impedance Z_S is constant.

- The flux under test conditions is the same as that under load conditions.

It is assumed that a given value of the field current always produces the same flux. This assumption introduces considerable error. When the armature is short circuited, the current in the armature lag the generated voltage by almost 90 degrees, and hence the armature reaction is almost completely demagnetizing.

- The effect of the armature reaction flux can be replaced by a voltage drop proportional to the armature current and that the armature reaction voltage drop is added to the armature reactance voltage drop.
- The magnetic reluctance to the armature flux is constant regardless of the power factor.

For a cylindrical rotor machine, this assumption is substantially true because of the uniform air gap. Regulation obtained by using a synchronous impedance method is higher than that obtained by actual loading. Hence, this method is also called the Pessimistic method.

At lower excitations, Z_S is constant, since the open circuit characteristics coincide with the air gap line. This value of Z_S is called the linear or Unsaturated Synchronous Impedance. However, with increasing excitation, the effect of saturation is to decrease Z_S and the values beyond the linear part of the open circuit called as Saturated Value of the Synchronous Impedance.

MMF Method of Voltage Regulation

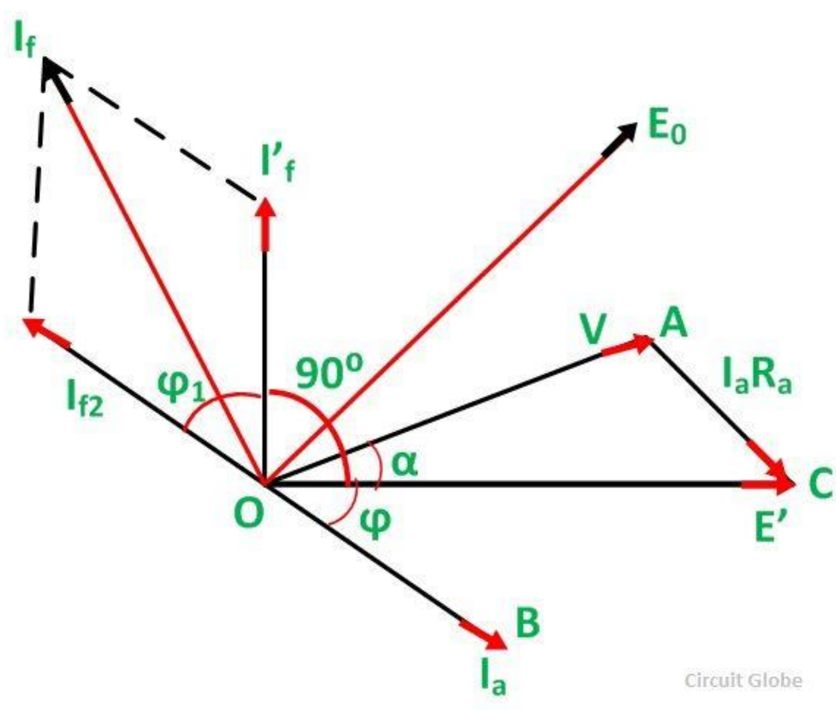
MMF Method is also known as Ampere Turn Method. The synchronous impedance method is based on the concept of replacing the effect of armature reaction by an imaginary reactance the Magnetomotive force (MMF). The MMF method replaces the effect of armature leakage reactance by an equivalent additional armature reaction MMF so that this MMF may be combined with the armature reaction MMF.

To calculate the voltage regulation by MMF Method, the following information is required. They are as follows:

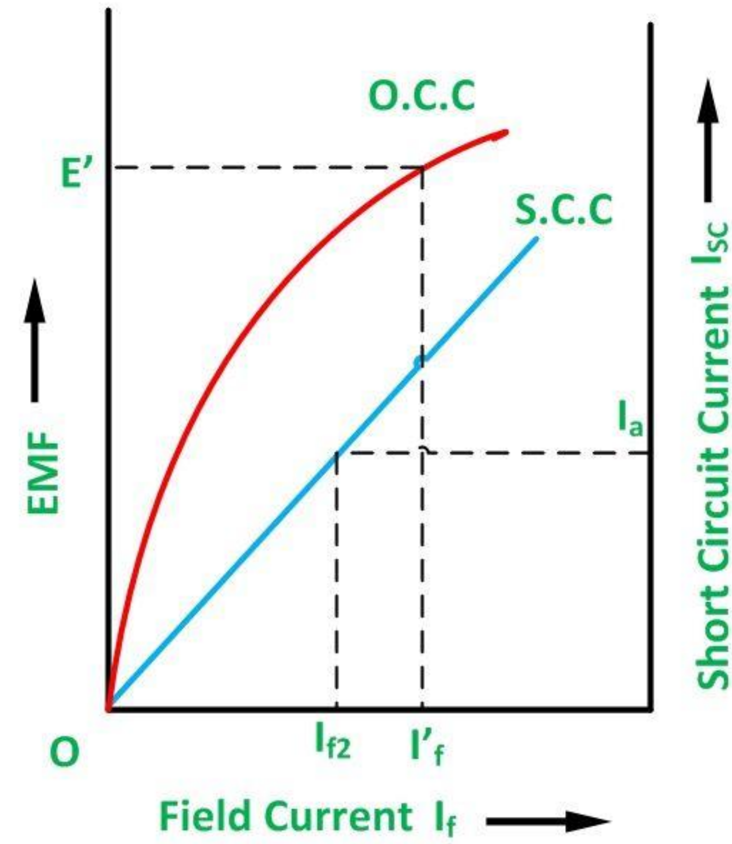
- The resistance of the stator winding per phase
- Open circuit characteristics at synchronous speed.
- Short circuit characteristic

Step to Draw Phasor Diagram of MMF Method

The phasor diagram at a lagging power factor is shown below.



- The armature terminal voltage per phase (V) is taken as the reference phasor along OA.
- The armature current phasor I_a is drawn lagging the phasor voltage for lagging power factor angle ϕ for which the regulation is to be calculated.
- The armature resistance drop phasor I_aR_a is drawn in phase with I_a along the line AC. Join O and C. OC represents the emf E'.



- Considering the Open Current Characteristics shown above the field current I'_f corresponding to the voltage E' is calculated.
- Draw the field current I'_f leading the voltage E' by 90 degrees. It is assumed that on short circuit all the excitation is opposed by the MMF of armature reaction. Thus,

$$I'_f = I'_f \angle 90^\circ - \alpha$$

From the Short Circuit Current characteristics (SSC) shown above, determine the field current I_{f2} required to circulate the rated current on short circuit. This is the filed current required to overcome the synchronous reactance drop I_aX_a.

- Draw the field current I_{f2} in phase in opposition to the current armature current I_a. Thus,

$$I_{f2} = I_{f2} \angle 180^\circ - \varphi$$

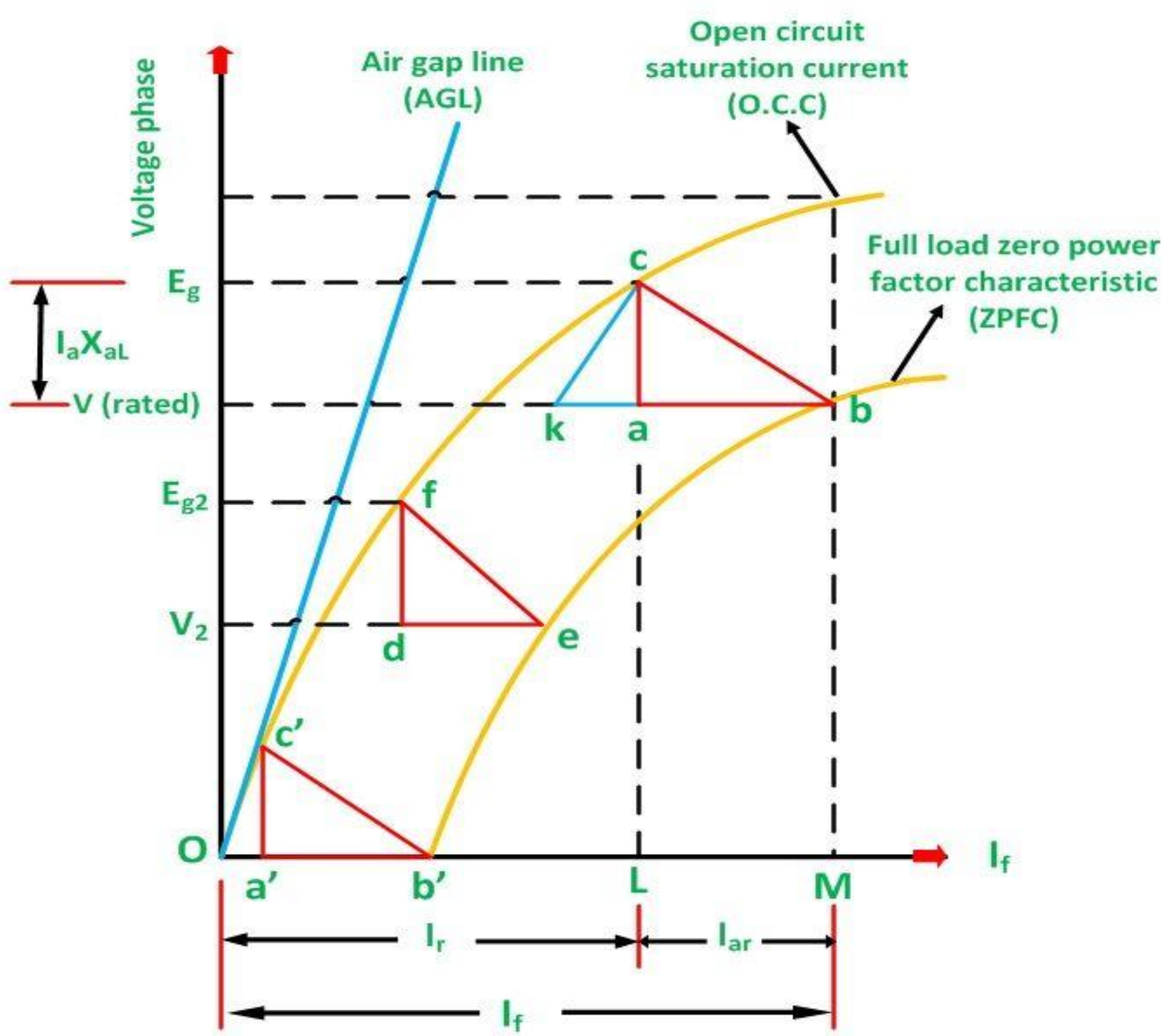
Determine the phasor sum of the field currents I'_f and I_{f2}. This gives the resultant field current If which would generate a voltage E_0 under no load conditions of the alternator. The open circuit EMF E_0 corresponding to the field current if is found from the open circuit characteristics

The regulation of the alternator is found from the relation shown below.

$$\text{Regulation} = \frac{E_0 - V}{V} \text{ x } 100\%$$

Potier Triangle or Zero Power Factor Method

The Potier triangle determines the voltage regulation of the machines. This method depends on the separation of the leakage reactance of armature and their effects. The graph of the Potier triangle is shown in the figure below. The triangle formed by the vertices a, b, c has shown below in the figure is called Potier Triangle.



Consider a point B on the Zero Power Factor Curve corresponding to rated terminal voltage V and a field current of OM = $I_f = F_f/T_f$. If, for this condition of operation the armature reaction MMF has a value expressed in equivalent field current will be given as

$$LM \left[= I_{ar} = \frac{F_{ar}}{T_f}\right]$$

Then the equivalent field current of the resultant MMF would be represented as shown below.

$$OL \left[= I_r = \frac{F_r}{T_f}\right]$$

This field current OL would result in a generated voltage $E_g = L_c$ from the no-load saturation curve. Since for lagging Zero Power Factor operation, the generated voltage will be

$$E_g = V + I_a X_{aL}$$

The vertical distance ac must be equal to the leakage reactance voltage DROP $I_a X_{aL}$ where I_a is the rated armature current.

Therefore,

$$X_{aL} = \frac{\text{Voltage ac per phase}}{\text{Rated armature current}}$$

For Zero Power Factor operation with rated current at any other terminal voltage, such as V_2. As the armature current is of the same value, both the $I_a$and X_{aL} voltage and the armature MMF must be of the same value. Therefore, for all the conditions of operation with rated armature current at zero lagging power factor, the Potier Triangle must be located between the terminal voltage V, a point on the ZPFC and the corresponding Eg point on the O.C.C.

If the Potier triangle cab is moved downward so that the side ab is kept horizontal and b is kept on the ZPFC, the point c will move on the O.C.C. When the point b, reaches the point e, the Potier triangle cab will move on the position fde shown in the figure. The location of point f on the O.C.C will determine the voltage Eg2. When the point b, reaches the point b', the Potier Triangle will be in the position c'a'b'. This is the limiting position which corresponds to short the circuit condition because the terminal voltage is zero at the point b'.

The initial part of the O.C.C is almost linear, another triangle Oc'b'is formed by the O.C.C. The hypotenuse of the Potier triangle and the baseline. A similar triangle such as ckb, can construct from the Potier triangle in any other location by drawing a line kc parallel to Oc'.

Steps for Construction of Potier Triangle on ZPFC

- Take a point b on the ZPFC preferably well upon the knee of the curve.

- Draw bk equal to b'O. (b' is the point for zero voltage, full load current). Ob' is the short circuit excitation Fsc.
- Through k draw, kc parallel to Oc' to meet O.C.C in c.
- Drop the perpendicular ca on to bk.
- Then, to scale ca is the leakage reactance drop $I_a X_{aL}$ and ab is the armature reaction MMF F_{aR} or the field current IfaR equivalent to armature reaction MMF at rated current.

The effect of field leakage flux in combination with the armature leakage flux gives rise to an equivalent leakage reactance Xp, known as the Potier Reactance. It is greater than the armature leakage reactance.

$$\text{Potier Reactance Xp} = \frac{\text{Voltage drop per phase which is equal to (ac)}}{\text{(ZPF rzted armature current per phase Ia)}}$$

For cylindrical rotor machines, the Potier reactance X_p is approximately equal to the leakage reactance X_{aL}. in salient pole machine, X_p may be as large as 3 times X_{aL}.

Assumptions for Potier Triangle

The following assumptions are made in the Potier Triangle Method. They are as follows:-

- The armature resistance R_a is neglected.
- The O.C.C taken on no load accurately represents the relation between MMF and Voltage on load.
- The leakage reactance voltage $I_a X_{aL}$ is independent of excitation.
- The armature reaction MMF is constant.

TWO REACTION THEORY OF SALIENT POLE MACHINE:

It is not necessary to plot the entire ZPFC for determining X_{aL} and F_a, only two points b and b' are sufficient. Point b corresponds to a field current which gives the rated terminal voltage while the ZPF load is adjusted to draw rated current. Point b' corresponds to the short circuit condition (V = 0) on the machine. Thus, Ob' is the field current required to circulate the short circuit current equal to the rated current.

In a Salient Pole Synchronous Machine Two Reaction Model the flux established by a mmf wave is independent of the spatial position of the wave axis with respect to the field pole axis. On the other hand, in a salient-pole machine as shown in the cross-sectional view of Fig., the permeance offered to a mmf wave is highest when it is aligned with the field pole axis (called the direct-axis or d-axis) and is lowest when it is oriented at 90° to the field pole axis (called the quadrature axis or q-axis).

Though the field winding in a salient-pole is of concentrated type, the B-wave produced by it is nearly sinusoidal because of the shaping of pole-shoes (the air-gap is least in the centre of the poles and progressively increases on moving away from the centre). Equivalently, the F_f waye can be imagined to be sinusoidally distributed and acting on a uniform air-gap. It can, therefore, be represented as space vector $\overline{F}_f$. As the rotor rotates, $\overline{F}_f$ is always oriented along the d-axis and is presented with the d-axis permeance. However in case of armature reaction the permeance presented to it is far higher when it is oriented along the d-axis then it is oriented along the q-axis.

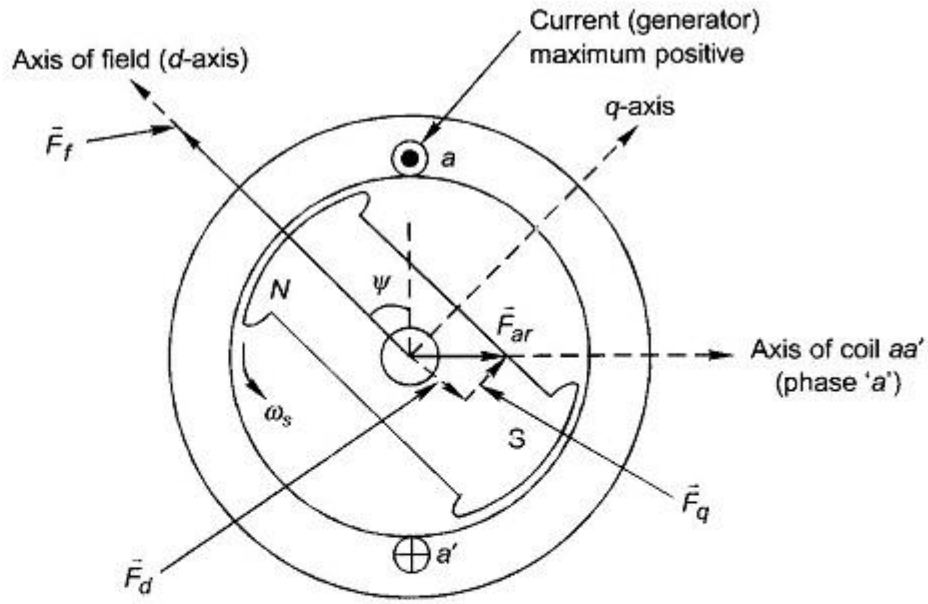


Figure shows the relative spatial location of $\overline{F}_f$ and $\overline{F}_{ar}$ at the time instant when current (generating) in phase a is maximum positive and is lagging $\overline{E}_f$. (excitation emf due to $\overline{F}_f$) by angle ψ As angle ψ varies, the permeance offered to $\overline{F}_{ar}$, the armature reaction mmf, varies because of a change in its spatial position relative to the d-axis. Consequently $\overline{F}_{ar}$ produces $\overline{\Phi}_{ar}$ (armature reactionflux/pole) whose magnitude varies with angle ψ (which has been seen earlier to be related to the load power factor). So long as the magnetic circuit is assumed linear (i.e. superposition holds), this difficulty can be overcome by dividing $\overline{F}_{ar}$ into vectors $\overline{F}_d$ along the d-axis and $\overline{F}_q$ along the q-axis as shown by dotted lines in Fig. 8.52.

Figure shows the phasor diagram corresponding to the vector diagram of Fig. 8.52. Here the d-axis is along $\overline{F}_f$ and 90° behind it is the q-axis along $\overline{E}_f$. The components of $\overline{I}_a$ and $\overline{F}_{ar}$ along d- and q-axis are shown in the figure from which it is easily observed, that $\overline{F}_d$ is produced by $\overline{I}_d$, the d-axis component of $\overline{I}_a$at 90° to $\overline{E}_f$ and $\overline{F}_q$ is produced by $\overline{I}_q$ the q-axis component of $\overline{I}_a$, in phase with $\overline{E}_f$

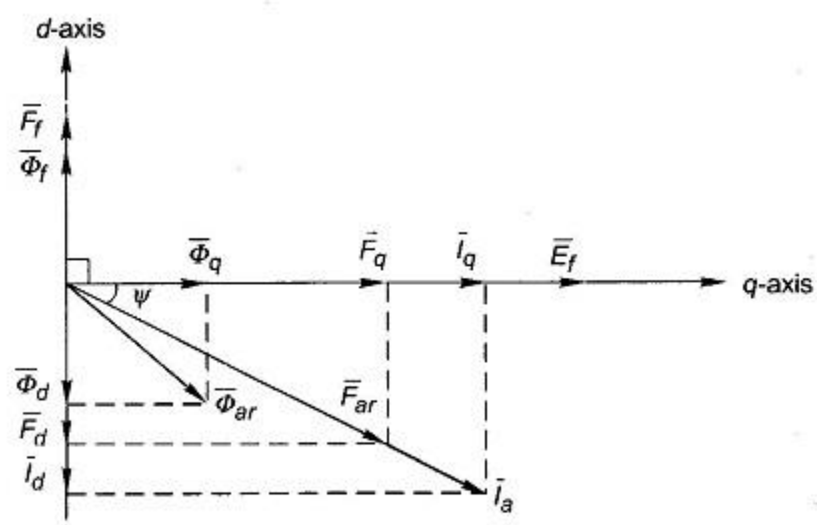
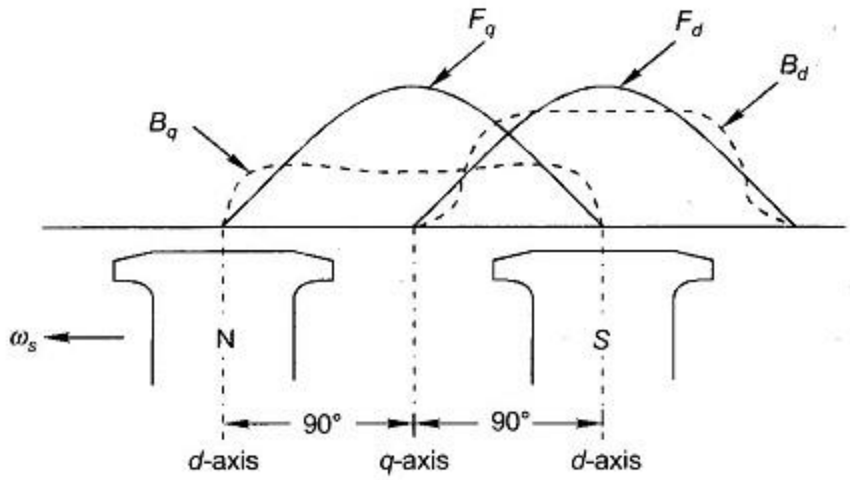


Figure shows the relative locations of F_d, F_q and field poles and the B-waves produced by these. It is easily seen that the B-waves contain strong third-harmonic space waves. For reasons advanced in Ch. 5, one can proceed with the analysis on the basis of space fundamentals of B_d and B_q while neglecting harmonics.*



The flux components/pole produced by the d- and q-axis components of armature reaction mmf are

$$\Phi_d = \mathcal{P}_d F_d = \mathcal{P}_d K_{ar} I_d \qquad \text{(in phase with } I_d\text{)}$$

$$\Phi_q = \mathcal{P}_q F_q = \mathcal{P}_q K_{ar} I_q \qquad \text{(in phase with } I_q\text{)}$$

where $\mathcal{P}_d$, $\mathcal{P}_q$ = permeance of pole-arc oriented along the *d*-axis/the *q*-axis ($\mathcal{P}_d > \mathcal{P}_q$).

The flux phasors $\bar{\Phi}_d$ and $\bar{\Phi}_q$ are also drawn in Fig. The resultant armature reaction flux phasor $\bar{\Phi}_{ar}$ is now no longer in phase with $\bar{F}_{ar}$ or $\bar{I}_a$ because $P_d > P_q$ in Eqs In fact ($\bar{\Phi}_{ar}$ lags or leads $\bar{I}_a$ depending upon the relative magnitudes of the d-axis, q-axis permeances.

The emfs induced by Φ_d and Φ_q are given by

$$\bar{E}_d = -jK_e\bar{\Phi}_d$$

$$\bar{E}_q = -jKe\bar{\Phi}_q$$

where K_e = emf constant of armature winding

The resultant emf induced in the machine is then

$$\bar{E}_r = \bar{E}_f + \bar{E}_d + \bar{E}_q$$

$$= \bar{E}_f - jK_e\bar{\Phi}_d - jK_e\bar{\Phi}_q$$

Substituting for $\bar{\Phi}_d$ and $\bar{\Phi}_q$ from Eqs

$$\bar{E}_r = \bar{E}_f - jK_e\mathcal{P}_dK_{ar}\bar{I}_d - jK_e\mathcal{P}_qK_{ar}\bar{I}_q$$

Let $X_d^{ar} = K_e\mathcal{P}_dK_{ar}$ = reactance equivalent of the d-axis component of armature reaction

$X_q^{ar} = K_e\mathcal{P}_qK_{ar}$ = reactance equivalent of the q-axis component of armature reaction

Obviously

$$X_d^{ar} > X_q^{ar} \qquad (\because \mathcal{P}_d > \mathcal{P}_q)$$

It then follows from Eq. (8.67) that

$$\bar{E}_f = \bar{E}_r + jX_d^{ar}\bar{I}_d + jX_q^{ar}\bar{I}_q$$

Also for a realistic machine

$$\bar{E}_r = \bar{V}_t + R_a\bar{I}_a + jX_l\bar{I}_a; \qquad \bar{I}_a = \bar{I}_d + \bar{I}_q$$

Combining Eqs

$$\bar{E}_f = \bar{V}_t + R_a\bar{I}_a + j(X_d^{ar} + X_l)\bar{I}_d + j(X_q^{ar} + X_l)\bar{I}_q$$

Define $X_d^{ar} + X_l = X_d$ = d-axis synchronous reactance

$X_q^{ar} + X_l = X_q$ = q-axis synchronous reactance

It is easily seen that

$$X_d > X_a$$

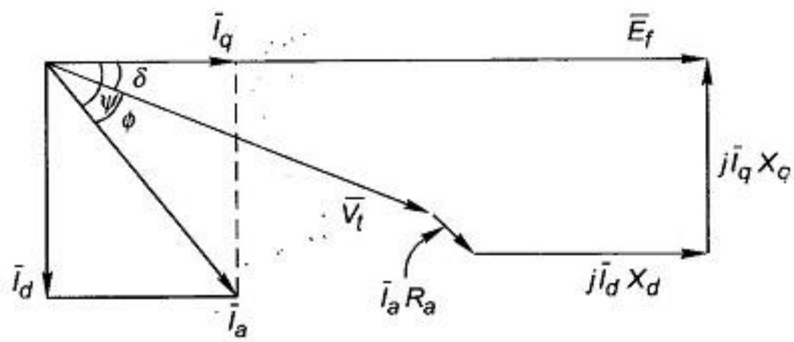
Equation can now be written as

$$\bar{E}_f = \bar{V}_t + R_a\bar{I}_a + jX_d\bar{I}_d + jX_q\bar{I}_q$$

The phasor diagram depicting currents and voltages as per Eqis drawn in Fig. in which δ is the angle between the excitation emf E_f and the terminal voltage V_t.

Analysis of Phasor Diagram

In the phasor diagram of Fig. 8.55, the angle ψ=Φ+δ is not known for a given V_t, I_a and Φ. The location of E_f being unknown, I_d and l_q cannot be found which are needed to draw the phasor diagram. This difficulty is overcome by establishing certain geometric relationships for the phasor diagrams which are drawn in Fig. for the generating machine and in Fig. for the motoring machine.

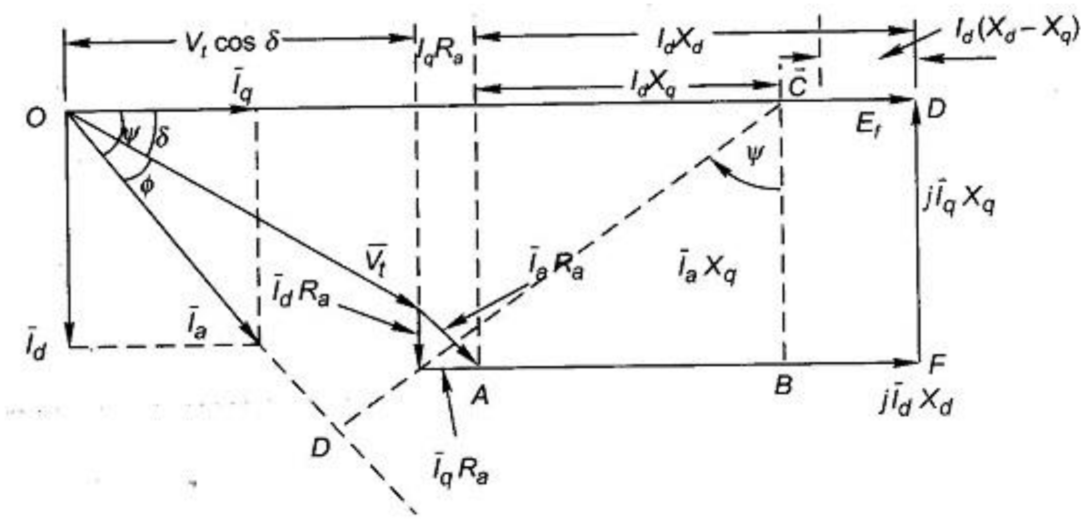


In Fig AC is drawn at 90° to the current phasor $\overline{I}_a$ and CB is drawn at 90° to $\overline{E}_f$. Now

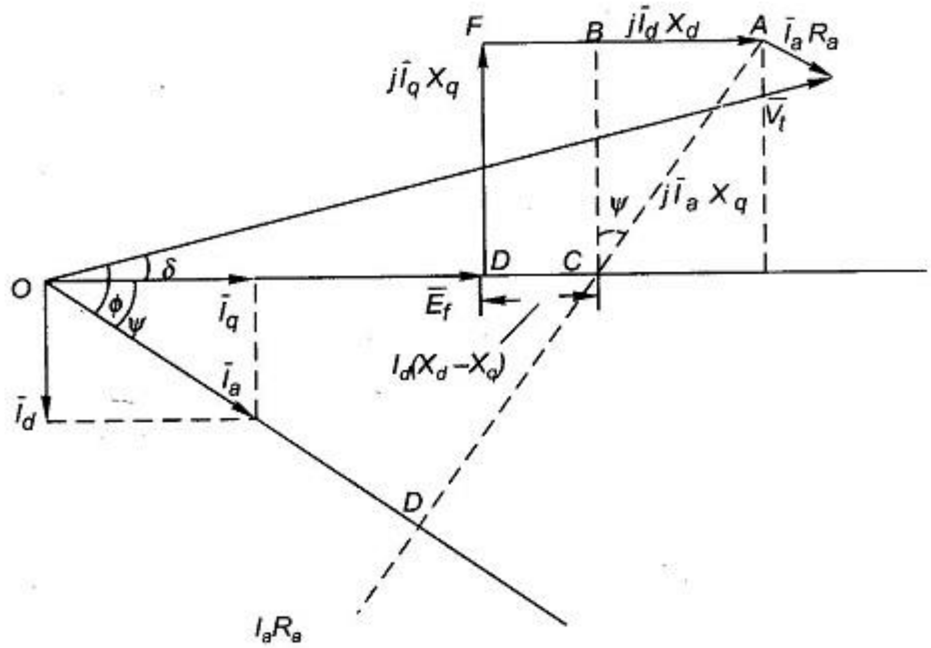
$$I_d = I_a \sin \psi$$

$$I_q = I_a \cos \psi$$

$$I_a = I_q / \cos \psi$$



(a) Generating, lagging pf



(b) Motoring, lagging pf

Fig. Geometric relationships for phasor diagram of salient-pole machine In AABC

In ΔABC

$$\cos\psi = \frac{BC}{AC} = \frac{I_q X_q}{AC}$$

or

$$AC = \frac{I_q X_q}{\cos\psi} = I_a X_q$$

It is easily seen that

$$AB = I_d X_q$$

and

$$CD = BF = I_d (X_d - X_q)$$

The phasor $\bar{E}_f$ can then be obtained by extending OC by +CD for generating machine and -CD for motoring machine, where CD is given by Eq.. Let us indicate

phasor OC by $\bar{E}'$.

Then in terms of magnitude

$$E_f = E' + I_d (X_d - X_q); \quad \begin{array}{l} + \text{ for generating machine,} \\ - \text{ for motoring machine} \end{array}$$

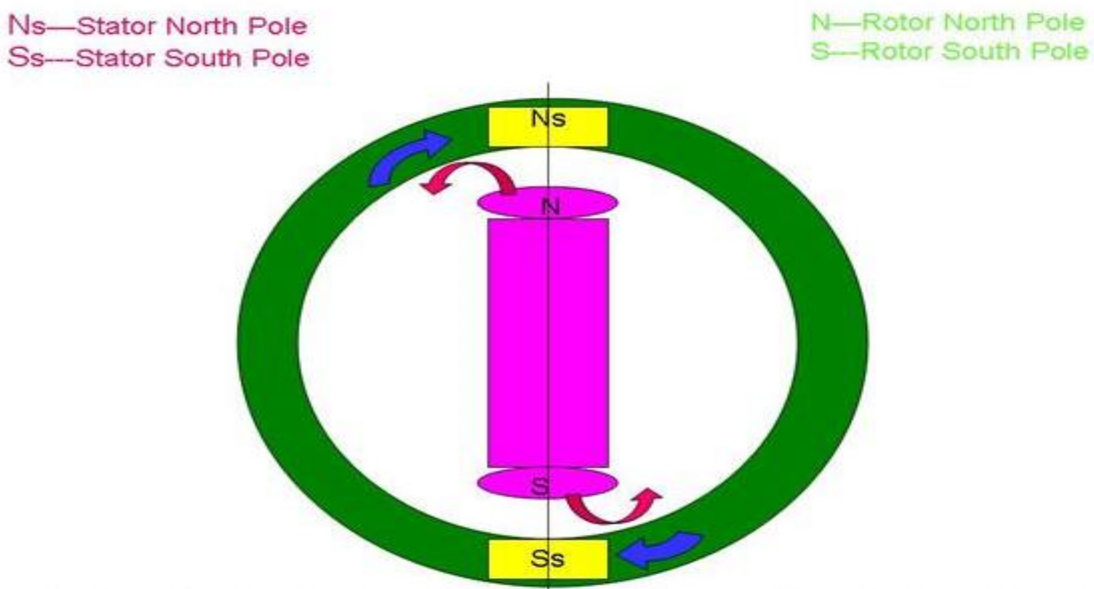
UNIT-V

SYNCHRONOUS MOTORS

Synchronous Motor Working Principle

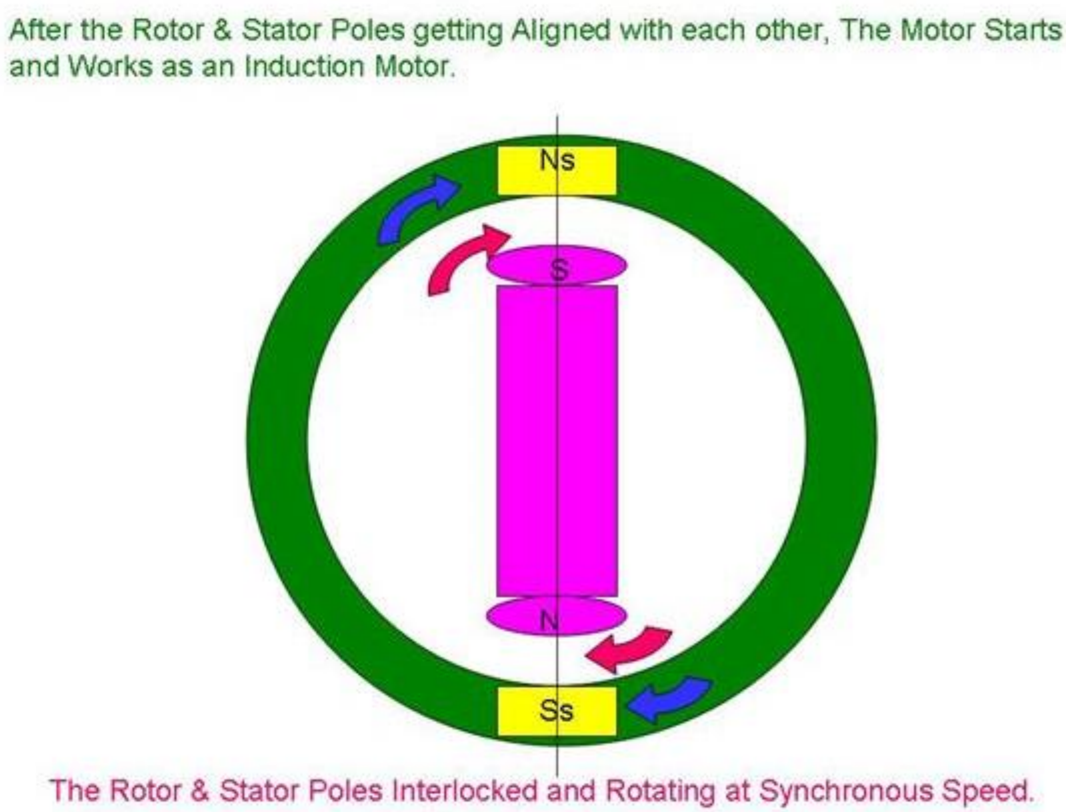
We have learnt about the various types of electric motors in our previous article. Now we will start to learn about these motors individually. In this article we will take a look at the synchronous motor theory of operation and its working.

The main principle is same as applicable for all motors. It is the mutual induction between the stator & rotor windings which make any motor operational. *Also when a 3 phase winding is fed with a 3 phase supply, then a magnetic flux of constant magnitude but rotating at Synchronous speed is produced.*



The Rotor & Stator Poles Repel each other, till they get Aligned with the Opposite Poles

To understand the working of a Synchronous Motor easily, let us consider only two poles in the stator and rotor. With reference to the figure, the stator has two poles Ns & Ss. These poles when energised, produces a rotating magnetic field, which can be assumed that the poles themselves are rotating in a circular manner. They rotate at a synchronous speed and let us assume the direction of rotation to be clockwise. If the rotor poles are at the position shown in the figure, we all know that *"Like poles repel each other"*. So, the North Pole in the stator repels the north pole of the rotor. Also the south pole of the stator repels the south of the rotor. This makes the rotor to rotate in anti-clockwise direction thus, half a period later, the stator poles interchange themselves, thus making them get aligned with *"unlike poles" which attracts each other*. I.e. the South Pole of the stator & the North Pole of the rotor gets attracted and get magnetically interlocked.



When the motor is supplied with a.c. power supply, the stator poles get energised. This in turn attracts (opposite) the rotor poles, thus both the stator and rotor poles get magnetically interlocked. It is this interlock which makes the rotor to rotate at the same synchronous speed with the stator poles. The synchronous speed of rotation is given by the expression Ns=120f/P.

When the load on the motor is increased gradually, the rotor even though runs at same speed, tends to progressively fall back in phase by some angle, "β", called the Load Angle or the Coupling Angle. This Load angle is dependent on the amount of load that the motor is designed to handle. In other words, we can interpret as the torque developed by the motor depends on the load angle, "β".

Starting procedure

All the Synchronous Motors are equipped with "Squirrel Cage winding", consisting of Cu (copper) bars, short-circuited at both ends. These windings also serve the purpose of self-starting of the Synchronous motor. During starting, it readily starts and acts as induction motors. For starting a Synchronous motor, the line voltage is applied to the stator terminals with the field terminals (rotor) left unexcited. It starts as an induction motor, and when it reaches a speed of about 95% of its synchronous speed, a weak d.c excitation is given to the rotor, thus making the rotor to align in synchronism with the stator.(at this moment the stator & rotor poles get interlocked with each other & hence pull the motor into synchronism.

Effect of Excitation on Armature Current and Power Factor

The value of excitation for which back e.m.f. Eb is equal (in magnitude) to applied voltage V is known as 100% excitation. We will now discuss what happens when motor is either over-excited or under-exicted although we have already touched this point

Consider a synchronous motor in which the mechanical load is constant (and hence output is also constant if losses are neglected).

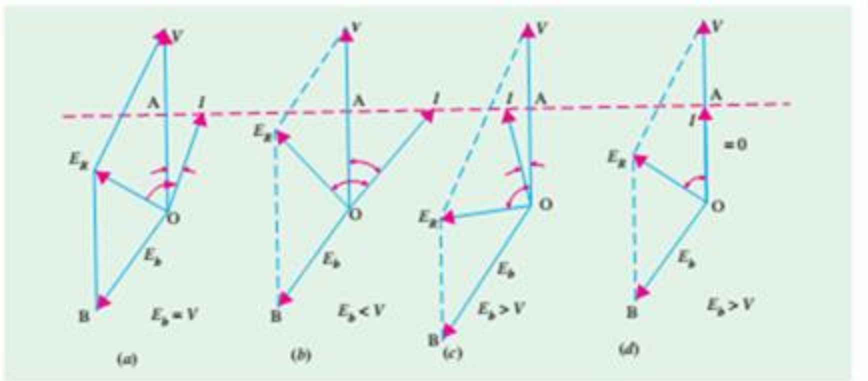


Fig. (a) shows the case for 100% excitation i.e., when Eb = V . The armature current I lags behind V by a small angle f. Its angle q with ER is fixed by stator constants i.e. tan q = X S / Ra.

In Fig. (b)* excitation is less than 100% i.e., Eb < V. Here, ER is advanced clockwise and so is armature current (because it lags behind ER by fixed angle q). We note that the magnitude of I is increased but its power factor is decreased (f has increased). Because input as well as V are constant, hence the power component of I i.e., I cos f remains the same as before, but wattless component I sin f is increased. Hence, as excitation is decreased, I will increase but p.f. will decrease so that power component of I i.e., I cos f = OA will remain constant. In fact, the locus of the extremity of current vector would be a straight horizontal line as shown.

Incidentally, it may be noted that when field current is reduced, the motor pull-out torque is also reduced in proportion.

Fig. (c) represents the condition for overexcited motor i.e. when Eb > V . Here, the resultant voltage vector ER is pulled anticlockwise and so is I. It is seen that now motor is drawing a leading current. It may also happen for some value of excitation, that I may be in phase with V i.e., p.f. is unity

[Fig. (d)]. At that time, the current drawn by the motor would be minimum.

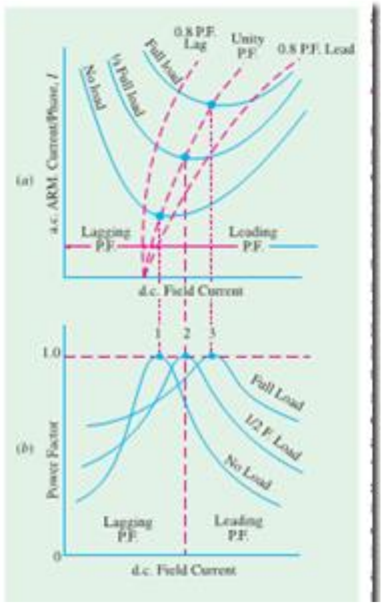
Two important points stand out clearly from the above discussion :

(i) The magnitude of armature current varies with excitation. The current has large value both for low and high values of excitation (though it is lagging for low excitation and leading for higher excitation). In between, it has minimum value corresponding to a certain excitation. The variations of I with excitation are shown in Fig. which are known as 'V ' curves because of their shape.

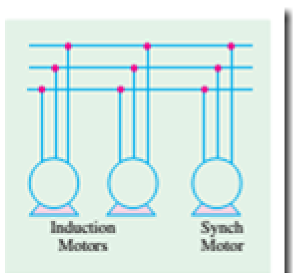
(ii) For the same input, armature current varies over a wide range and so causes the power factor also to vary accordingly. When over-excited, motor runs with leading p.f. and with lagging

p.f. when under-excited. In between, the p.f. is unity. The variations of p.f. with excitation

are shown in Fig. The curve for p.f. looks like inverted 'V ' curve. It would be noted that minimum armature current corresponds to unity power factor.



It is seen that an over-excited motor can be run with leading power factor. This property of the motor renders it extremely useful for phase advancing (and so power factor correcting) purposes in the case of industrial loads driven by induction motors (Fig) and lighting and heating loads supplied through transformers. Both transformers and induction motors draw lagging currents from the line. Especially on light loads, the power drawn by them has a large reactive component and the power factor has a very low value. This reactive component, though essential for operating the electric m a c h i n e entails appreciable loss in many ways. By using synchronous motors in conjunction with induction motors and transformers, the lagging reactive power required by the latter is supplied locally by the leading reactive component taken by the former, thereby relieving the line and generators of much of the reactive component. Hence, they now supply only the active component of the load current. When used in this way, a synchronous motor is called a synchronous capacitor, because it draws, like a capacitor, leading current from the line. Most synchronous capacitors are rated between 20 MVAR and 200 MVAR and many are hydrogen-cooled.

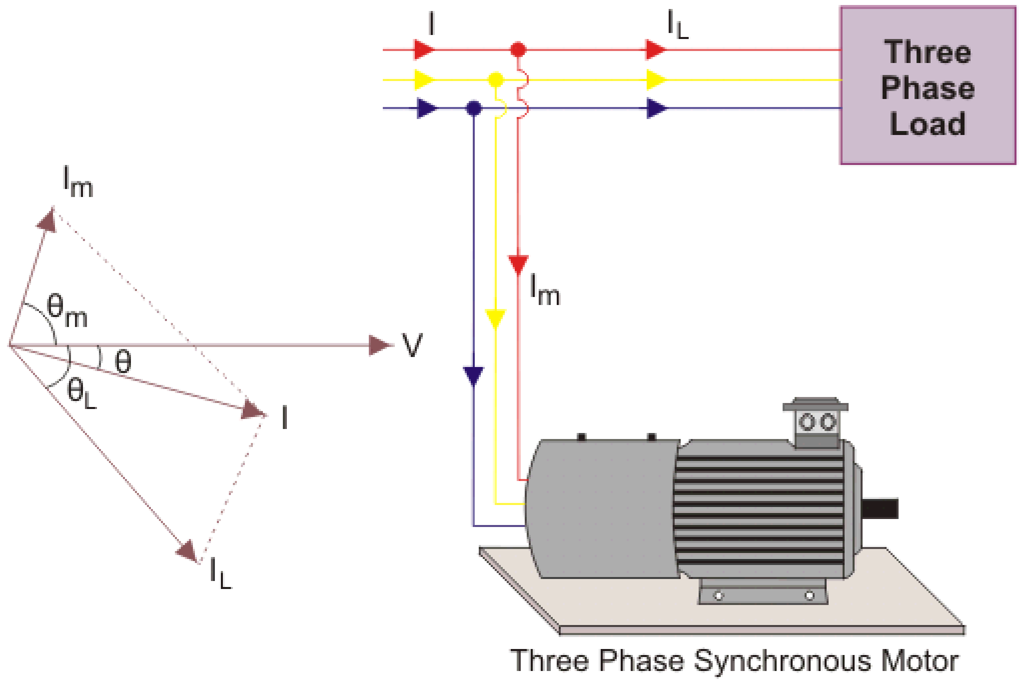


Synchronous Condenser

Like capacitor bank, we can use an overexcited synchronous motor to improve the poor power factor of a power system. The main advantage of using synchronous motor is that the improvement of power factor is smooth.

When a synchronous motor runs with over-excitation, it draws leading current from the source. We use this property of a synchronous motor for the purpose.

Here, in a three-phase system, we connect one three-phase synchronous motor and run it at no load.



Suppose due to a reactive load of the power system the system draws a current I_L from the source at a lagging angle θ_L in respect of voltage. Now the motor draws a I_M from the same source at a leading angle θ_M. Now the total current drawn from the source is the vector sum of the load current I_L and motor current I_M. The resultant current I drawn from the source has an angle θ in respect of voltage. The angle θ is less than angle θ_L. Hence power factor of the system $\cos\theta$ is now more than the power factor $\cos\theta L$ of the system before we connect the synchronous condenser to the system.

The synchronous condenser is the more advanced technique of improving power factor than a static capacitor bank, but power factor improvement by synchronous condenser below 500 kVAR is not economical than that by a static capacitor bank. For major power network we use synchronous condensers for the purpose, but for comparatively lower rated systems we usually employ capacitor bank.

The advantages of a synchronous condenser are that we can control the power factor of system smoothly without stepping as per requirement. In case of a static capacitor bank, this fine adjustments of power factor cannot be possible rather a capacitor bank improves the power factor stepwise. The short circuit withstand-limit of the armature winding of a synchronous motor is high.

Although, synchronous condenser system has some disadvantages. The system is not silent since the synchronous motor has to rotate continuously. An ideal load less synchronous motor draws leading current at 90^o(electrical).

Hunting in Synchronous Motor

We come across the term HUNTING when we study about three phase synchronous motor operations. The word hunting is used because after sudden application of load the rotor has to search or hunt for its new equilibrium position. That phenomenon is referred to as hunting in synchronous motor. Now let us know what is the condition of equilibrium in synchronous motor. A steady state operation of synchronous motor is a condition of equilibrium in which the electromagnetic torque is equal and opposite to load torque. In steady state, rotor runs at synchronous speed thereby maintaining constant value of torque angle (δ). If there is sudden change in load torque, the equilibrium is disturbed and there is resulting torque which changes speed of the motor.

What is Hunting?

Unloaded synchronous machine has zero degree load angle. On increasing the shaft load gradually load angle will increase. Let us consider that load P_1 is applied suddenly to unloaded machine shaft so machine will slow down momentarily. Also load angle (δ) increases from zero degree and becomes δ_1. During the first swing electrical power developed is equal to mechanical load P_1. Equilibrium is not established so rotor swings further. Load angle exceeds δ_1 and becomes δ_2. Now electrical power generated is greater than the previous one. Rotor attains synchronous speed. But it does not stay in synchronous speed and it will continue to increase beyond synchronous speed. As a result of rotor acceleration above synchronous speed the load angle decreases. So once again no equilibrium is attained. Thus rotor swings or oscillates about new equilibrium position. This phenomenon is known as hunting or phase swinging. Hunting occurs not only in synchronous motors but also in synchronous generators upon abrupt change in load.

Causes of Hunting in Synchronous Motor

1. Sudden change in load.
2. Sudden change in field current.
3. A load containing harmonic torque.
4. Fault in supply system.

Effects of Hunting in Synchronous Motor

1. It may lead to loss of synchronism.
2. Produces mechanical stresses in the rotor shaft.
3. Increases machine losses and cause temperature rise.
4. Cause greater surges in current and power flow.
5. It increases possibility of resonance.

Reduction of Hunting in Synchronous Motor

Two techniques should be used to reduce hunting. These are –

- Use of Damper Winding: It consists of low electrical resistance copper / aluminum brush embedded in slots of pole faces in salient pole machine. Damper winding damps out hunting by producing torque opposite to slip of rotor. The magnitude of damping torque is proportional to the slip speed.

- Use of Flywheels: The prime mover is provided with a large and heavy flywheel. This increases the inertia of prime mover and helps in maintaining the rotor speed constant.

- Designing synchronous machine with suitable synchronizing power coefficients.

METHODS OF STARTING OF SYNCHRONOUS MOTOR:

As seen earlier, synchronous motor is not self starting. It is necessary to rotate the rotor at a speed very near to synchronous speed. This is possible by various method in practice. The various methods to start the synchronous motor are,

1. Using pony motors
2. Using damper winding
3. As a slip ring induction motor
4. Using small d.c. machine coupled to it.

1. Using pony motors

In this method, the rotor is brought to the synchronous speed with the help of some external device like small induction motor. Such an external device is called 'pony motor'.

Once the rotor attains the synchronous speed, the d.c. excitation to the rotor is switched on. Once the synchronism is established pony motor is decoupled. The motor then continues to rotate as synchronous motor.

2. Using Damper Winding

In a synchronous motor, in addition to the normal field winding, the additional winding consisting of copper bars placed in the slots in the pole faces. The bars are short circuited with the help of end rings. Such an additional winding on the rotor is called damper winding. This winding as short circuited, acts as a squirrel cage rotor winding of an induction motor. The schematic representation of such damper winding is shown in the Fig.

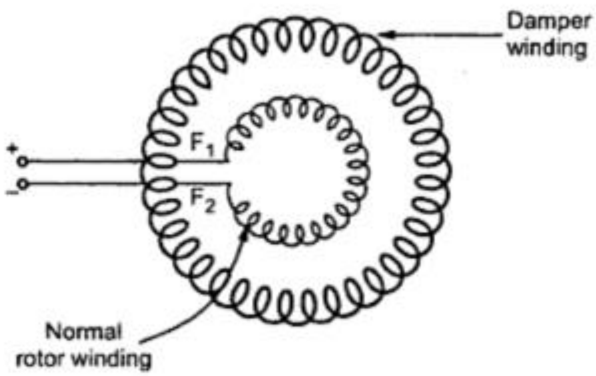


Fig . Starting as a squirrel cage I.M.

Once the rotor is excited by a three phase supply, the motors starts rotating as an induction motor at sub synchronous speed. Then d.c. supply is given to the field winding. At a particular instant motor gets pulled into synchronism and starts rotating at a synchronous speed. As rotor rotates at synchronous speed, the relative motion between damper winding and the rotating magnetic field is zero. Hence when motor is running as synchronous motor, there can not be any induced e.m.f. in the damper winding. So damper winding is active only at start, to run the motor as an induction motor at start. Afterwards it is out of the circuit. As damper winding is short circuited and motor gets started as induction motor, it draws high current at start so induction motor starters like star-delta, autotransformer etc. used to start the synchronous motor as an induction motor.

3. As a Slip Ring Induction Motor

The above method of starting synchronous motor as a squirrel cage induction motor does not provide high starting torque. So to achieve this, instead of shorting the damper winding, it is designed to a form a three phase star or delta connected winding. The three ends of this winding are brought out through slip rings. An external rheostat then can be introduced in series with the rotor circuit. So when stator is excited, the motor starts as a slip ring induction motor and due to resistance added in the rotor provides high starting torque. The resistance is then gradually cut off, as motor gathers speed. When motor attains speed near synchronous. d.c. excitation is provided to the rotor, then motors gets pulled into synchronism ans starts rotating at synchronous speed. The damper winding is shorted by shorting the slip rings. The initial resistance added in the rotor not only provides high starting torque but also limits high inrush of starting current. Hence it acts as a motor resistance starter.

The synchronous motor started by this method is called a slip ring induction motor is shown in the Fig.

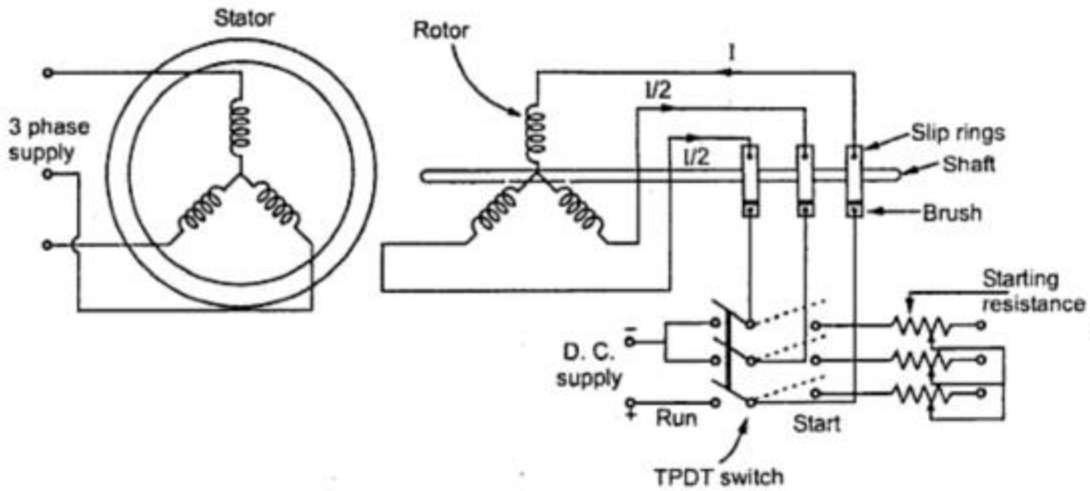


Fig. Starting as a slip ring I.M.

It can be observed from the Fig. that the same three phase rotor winding acts as a normal rotor winding by shorting two of the phases. From the positive terminal, current 'I' flows in one of the phases, which divides into two other phases at start point as 1/2 through each, when switch is thrown on d.c. supply side.

4. Using Small D.C. Machine

Many a times, a large synchronous motor are provided with a coupled d.c. machine. This machine is used as a d.c. motor to rotate the synchronous motor at a synchronous speed. Then the excitation to the rotor is provided. Once motor starts running as a synchronous motor, the same d.c. machine acts as a d.c. generator called exciter. The field of the synchronous motor is then excited by this exciter itself.

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